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# Numerical solution of Cauchy problems in linear elasticity in axisymmetric situations

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ABSTRACT

An iterative method for solving axisymmetric Cauchy problems in linear elasticity is presented. This kind of problem consists in recovering missing displacements and forces data on one part of a domain boundary from the knowledge of overspecified displacements and forces data on another part of this boundary. Numerical simulations using the finite element method highlight the algorithm's efficiency, accuracy and robustness to noisy data as well as its ability to deblur noisy data. An application of the inverse technique to the identification of a friction coefficient is also presented.

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## 1. Introduction

Inverse problems can be defined by opposition to direct problems (Kubo, 1988) and characterized by the lack of knowledge of one of the following elements of information: the geometry of the domain, the equilibrium equations, the constitutive equations, the boundary conditions on the whole boundary of the domain and the initial conditions. According to this definition, many mechanical problems, for instance, identification of material parameters, identification of unknown boundaries (such as cracks or cavities), identification of initial boundary conditions, identification of inaccessible boundary conditions can be considered as inverse problems and more specific examples relating to elasticity problems can be found in Bonnet and Constantinescu (2005).

In a mathematical sense, direct problems can be considered as well-posed problems. In linear cases, these problems have a unique solution which is stable (continuously dependent on the data). Conversely, inverse problems are generally ill-posed problems in the Hadamard sense (Hadamard, 1923), since the existence or uniqueness or the continuous dependence on the data of their solutions may not be ensured.

This paper examines, in axisymmetric situations, an inverse boundary value problem in linear elasticity, namely known as a Cauchy problem. It consists in recovering missing displacement and force data on some part of the boundary of a domain from overspecified displacement and force data on another part. In this case, the equilibrium equations, the constitutive equations, the domain and its boundary are known.

In order to solve Cauchy problems in linear elasticity, many regularization methods have been introduced which can be classified as Tikhonov type methods (Bilotta and Turco, 2009; Koya et al., 1993; Maniatty et al., 1989; Marin and Lesnic, 2002a, 2003, 2004; Marin, 2005; Schnur and Zabaras, 1990; Tikhonov and Arsenin, 1977; Yeih et al., 1993; Zabaras et al., 1989) or iterative methods (Andrieux and Baranger, 2008; Delvare et al., 2010; Ellabib and Nachaoui, 2008; Marin, 2009; Marin et al., 2001, 2002b; Marin and Lesnic, 2005; Marin and Johansson, 2010a,b) Tikhonov regularization methods present the advantage of leading to well-posed problems where the equilibrium equations have been modified. Some iterative methods are based on the use of a sequence of well-posed problems and others on the minimization of an energy-like functional. Numerical algorithms are implemented using different numerical methods, such as the finite element method (FEM) (Andrieux and Baranger, 2008; Bilotta and Turco, 2009; Delvare et al., 2010; Maniatty et al., 1989; Schnur and Zabaras, 1990), the boundary element method (BEM) (Ellabib and Nachaoui, 2008; Koya et al., 1993; Marin et al., 2001, 2002a,b; Marin and Lesnic, 2002a,b, 2003, 2005, 2010a; Marin, 2009; Yeih et al., 1993; Zabaras et al., 1989) or meshless methods (Marin and Lesnic, 2004; Marin, 2005; Marin and Johansson, 2010b). Some papers present comparisons between different numerical methods (Marin et al., 2002a; Marin, 2009).

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