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The torsional buckling of a cruciform column under compressive load with a vertex plasticity model

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ABSTRACT

The torsional buckling of a plastically deforming cruciform column under compressive load is investigated. The problem is solved analytically based on the von Kármán shallow shell theory and the virtual work principle. Solutions found in the literature are extended for path-dependent incremental behaviour as typically found in the presence of the vertex effect that is present in metallic polycrystals.

At the critical load for buckling the direction of straining changes by an additional shear component. It is shown that the incremental elastic–plastic moduli are spatially nonuniform for such situations, contrary to the classical J_2 flow and deformation theories. The critical shear modulus that governs the buckling equation is obtained as a weighted average of the incremental elastic–plastic moduli over the cross-section of the cruciform.

Using a plasticity model proposed by the authors, that includes the vertex effect, the buckling-critical load is computed for a aluminium column both with the analytical model and a FEM-based eigenvalue buckling analysis. The stable post-buckling path is determined by the energy criterion of path-stability. A comparison with the experimentally obtained classical results by Gerard and Becker (1957) shows good agreement without relying on artificial imperfections as necessary in the classical J_2 flow theory.

1. Introduction

Thin walled open structures tend to buckle in the torsion mode under applied compressive loads. If the applied load exceeds the yield load, the twisting structure remains in the plastic state in the whole cross-section. This is in contrast to slender columns that buckle in a bending mode. Here, part of the cross-section unloads.

Moreover, bending is a completely one-dimensional case without any influence of the vertex effect. In the torsion mode, however, the flanges of the column show additional twisting and thus change from compression to a combination of compression and shear.

The plastic cruciform is an example for the failure of J_2 flow theory in prediction of buckling loads of perfect structures (Batdorf, 1949) also known as the plastic buckling paradoxon. Today it is well documented in many textbooks, e.g. Nguyen (2000, sect. 14.3.2.2).

Historically, Stowell (1948) first presented a unified theory of plastic buckling of columns and plates. Stowell (1951) also compared experimental buckling investigations for cruciform columns with analytical results obtained using the the deformation theory and proved a match. Finally, Gerard and Becker (1957) compared experimental results for cruciform columns made out of 2024-T4 Aluminum alloys with different analytical approaches. Various ratios of flange width and thickness were tested. The J_2 flow theory of plasticity and the deformation theory have been used to obtain analytical results. The latter appeared to favor the deformation theory. These results gave experimental evidence for certain shortcomings of the J_2 flow theory in calculation of buckling loads for the first time (Caner et al., 2002).

The apparent advantage of the deformation theory has been explained by the similarity to the slip theory first proposed by Batdorf and Budiansky (1949). The latter predicts a vertex in the yield surface at the current stress point. For total loading histories, integration of distinct slip theories is possible leading to deformation theories of plasticity (Sanders, 1954; Hutchinson, 1974; Stören and Rice, 1975).

When directly comparing J_2 flow and deformation theory for prediction of buckling loads, experimental and analytical work often favour the deformation theory (Tugcu, 1991; Blachut et al., 1996; Wang et al., 2001). For buckling of circular tubes, Bardi et al. (2006), Corona et al. (2006) used the incremental moduli derived from the deformation theory as a linear comparison solid in the sense of Hill (1958) for computation of the buckling load under compressive and bending loads, respectively. The plastic buckling of different shell structures has been compared by Bushnell (1982).

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