



# A material model for cementitious composite materials with an exterior point Eshelby microcrack initiation criterion

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## ARTICLE INFO

### Article history:

Received 17 August 2010

Received in revised form 12 May 2011

Available online 12 August 2011

### Keywords:

Micromechanics

Microcrack

Concrete

Constitutive

Interface

## ABSTRACT

A micromechanical model for cementitious composite materials is described in which microcrack initiation, in the interfacial transition zone between aggregate particles and cement matrix, is governed by an exterior-point Eshelby solution. The model assumes a two-phase elastic composite, derived from an Eshelby solution and the Mori–Tanaka homogenization method, to which circular microcracks are added. A multi-component rough crack contact model is employed to simulate normal and shear behaviour of rough microcrack surfaces. The development of the microcrack initiation criterion and the rules adopted for microcrack evolution are a particular focus of the paper. Finally, it is shown, on the basis of several numerical simulations, that the model captures key characteristics of the behaviour of cementitious composites such as concrete.

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## 1. Introduction

Extensive research has been carried out over the last few decades in order to explain and model damage phenomena in quasi-brittle materials such as concrete. It is generally accepted that the heterogeneous structure of such materials observed at micro and meso levels determines their complex macroscopic behaviour and failure mechanisms (van Mier, 1997).

A number of macroscopic phenomenological models based on damage and plasticity theories have been formulated (e.g. Comi and Perego, 2001; di Prisco and Mazars, 1996; Este and Willam, 1994; Feenstra and de Borst, 1995; Lee and Fenves, 1998; Grassl et al., 2002; Grassl and Jirásek, 2006; Nguyen and Korsunsky, 2008), in which the plastic and damage internal variables have been assumed to be scalars, vectors or higher order tensors. Although their numerical implementation in finite element codes is sometimes relatively straightforward, they do not, in general, properly capture all of the physical mechanisms that control the complex behaviour of these materials and can often use parameters that are difficult to determine and which do not have physical meanings.

In recent years, several models that aim to capture macroscopic behaviour by simulating the physical mechanisms at micro and meso levels have been developed using a micromechanical approach. Pensée et al. (2002) and Pensée and Kondo (2003) formulated an anisotropic damage model by employing a micromechanical

solution for an elastic solid containing non-interacting penny-shaped microcracks and an energy release rate-based damage criterion that incorporates frictionless crack closure. Gambarotta (2004) proposed an anisotropic friction-damage model based on the solution of an elastic body containing plane cracks. Pichler et al. (2007) combined fracture energy theory and continuum micromechanics to formulate a damage evolution law in a tensile strain-softening model for brittle materials based on the propagation of interacting microcracks. Recently, Zhu et al. (2008, 2009) developed an anisotropic damage model using the classic Eshelby inclusion solution and a thermodynamics-based damage evolution law coupled with Coulomb friction sliding along closed crack surfaces. Unilateral effects as well as the interaction, shape and spatial distribution of microcracks were taken into account through a homogenization procedure based on the scheme proposed by Ponte-Castaneda and Willis (1995).

Micromechanical solutions have also been employed to develop effective models for the prediction of elastic and strength properties of cementitious materials (Pichler and Hellmich, 2011).

The Microplane model was originally inspired by micromechanics (Bazant and Prat, 1988) but was subsequently developed along a more phenomenological path (Bazant and Caner, 2005) and thus differs from the more mechanistic micromechanical models discussed above.

Jefferson and Bennett (2007, 2010) developed a micromechanical model that simulates a two-phase composite material containing randomly distributed penny-shaped microcracks which develop according to a local damage evolution function. The Mori–Tanaka homogenization method was adopted to account for the interaction between microcracks. Stress recovery across

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