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# Static behavior and bifurcation of a monosymmetric open cross-section thin-walled beam: Numerical and experimental analysis

## Angelo Di Egidio<sup>a,\*</sup>, Fabrizio Vestroni<sup>b</sup>

<sup>a</sup> Department of Structural, Hydraulic and Geotechnical Engineering, University of L'Aquila, L'Aquila, Italy <sup>b</sup> Department of Structural and Geotechnical Engineering, University of Rome "La Sapienza", Rome, Italy

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#### ABSTRACT

The aim of the paper is the numerical and experimental validation of a previously developed nonlinear one-dimensional model of inextensional, shear undeformable, thin-walled beam with an open cross-section. Nonlinear in-plane and out-of-plane warping and torsional elongation effects are included in the model. To better understand the role of these new contributions a beam with a section with one symmetry axis, undergoing moderately large flexural curvatures and large torsional curvature is taken into account. To obtain a section of a cantilever beam for which the torsional curvature is expected to prevail with respect to the flexural ones, a preliminary study is performed. The attention is focused on the response to static forces and on the stability of the equilibrium branches. Analytical results are compared with results of two different nonlinear finite element models and mainly with experimental results to confirm the validity of the analytical model. Interesting results are obtained for the critical values of the flexural-torsional instability loads.

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#### 1. Introduction

The description of the nonlinear behavior of beams in 3D has been the subject of a certain number of papers in the last decades. A one-dimensional polar model of a compact beam was initially studied in Crespo da Silva and Glynn (1978); the torsional component was statically condensed and warping was neglected. A threedimensional beam model was developed in Rosen and Friedmann (1979) for a compact cross-section beam by also taking into account the linear warping. In Crespo da Silva (1988, 1991) and Crespo da Silva and Zaretzky (1994) flexural-torsional-extensional couplings in the motion of a cantilever beam was considered, limiting the model to linear warping. Even if the linear warping contribution is considered, attention is generally paid to the case of compact cross-sections. In Luongo et al. (1989) a non-compact cross-section beam, having close bending and torsional frequencies was studied but due to difficulties in tackling nonlinear warping, it was preferred to completely neglect it. An approach based on the extension of the Vlasov theory (Vlasov, 1961) to the nonlinear field is found in Ghobarah and Tso (1971), Mollmann (1982) and Ascione and Feo (1995); however, due to the complexity of the problem, several simplifying assumptions have been used, i.e. flexural curvatures up-to linear terms are considered. In recent years similar problems have been studied in a numerical way by using the model developed in Schulz and Filippou (1998) where a non-uniform warping theory of bars employing two-dimensional St. Venant warping functions and one-dimensional independent warping parameters has been developed. In Sapountzakis and Mokos (2003) and Sapountzakis and Tsiatas (2007, 2008), by using boundary value problems through FEM, warping and shear stresses of bars under arbitrary loading have been evaluated, while static and dynamic analyses accounting of nonlinear torsional have been performed.

More recently in Di Egidio et al. (2003, Part I and II) the description of the mechanical behavior of beams with open cross-sections is dealt with by using a nonlinear beam model developed in a rigorous manner starting from an internally constrained threedimensional continuum, in which torsional and flexural curvatures of the same order of magnitude are considered. The warping is obtained by extending the Vlasov theory (Vlasov, 1961) to the nonlinear field. The effects of the torsional curvature on the elongation of the longitudinal fibers and the nonlinear warping of the section are considered. The model obtained is very complex and the equilibrium equations are derived by means of symbolic manipulation tools missing any possibility of recognizing the mechanical meaning of the different numerous terms. Within a class of open cross section beams with certain geometrical and mechanical characteristics, it can be shown that torsional curvature is greater than the flexural ones. A notable simplification is then obtained in kinematical relations with respect to the model developed in Di Egidio et al. (2003, Part I and II), as partially presented in Vestroni et al. (2006).

<sup>\*</sup> Corresponding author.

*E-mail addresses:* angelo.diegidio@univaq.it (A. Di Egidio), vestroni@uniroma1.it (F. Vestroni).