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Displacement discontinuity method for modeling axisymmetric cracks in an elastic half-space

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ABSTRACT

This paper describes a displacement discontinuity method for modeling axisymmetric cracks in an elastic half-space or full space. The formulation is based on hypersingular integral equations that relate displacement jumps and tractions along the crack. The integral kernels, which represent stress influence functions for ring dislocation dipoles, are derived from available axisymmetric dislocation solutions. The crack is discretized into constant-strength displacement discontinuity elements, where each element represents a slice of a cone. The influence integrals are evaluated using a combination of numerical integration and a recursive procedure that allows for explicit integration of hyper- and Cauchy singularities. The accuracy of the solution at the crack tip is ensured by adding corrective stresses across the tip element. The method is validated by a comparison with analytical and numerical reference solutions.

1. Introduction

The displacement discontinuity method (DDM), as originally formulated by Crouch (1976b), is a boundary element method that relies on distributing straight elements with constant displacement jumps along the boundary of the domain, including cracks, to solve plane elasticity problems. The DDM traces its roots to a method originally developed to analyze the elastic perturbations induced by mining tabular excavations (Berry, 1963; Salamon, 1963, 1964; Starfield and Crouch, 1973). The hypersingular integral equations that underlie the DDM were described by Bui (1977) among others and were first used directly by Ioakimidis (1982, 1983) and by Murakami and Nemat-Nasser (1982) to solve crack problems. These integral equations were reformulated in terms of complex variables by Linkov and Mogilevskaya (1994) and later applied to the solution of two-dimensional curvilinear crack problems by Mogilevskaya (1997, 2000). Further information on the historical developments of this method can be found elsewhere (Crouch, 1976a; Linkov and Mogilevskaya, 1998).

The hypersingular integral equations actually represent distributions of dislocation dipoles with density corresponding to the actual displacement discontinuities along the crack, and to fictitious displacement jumps along other boundaries of the elastic domain. The DDM is thus the result of a discretization of these equations with the boundary divided into segments and the displacement jumps along each segment assumed to be constant or more generally to vary according to a linear, quadratic, or cubic polynomial (e.g. Crawford and Curran, 1982; Napier and Malan, 1997; Peirce, 2010). A related method is to globally approximate the displacement discontinuity fields along the crack by the product of a function with a square root behavior at the tips and a truncated series of orthogonal polynomials (Korsunsky and Hills, 1995; Hills et al., 1996).

These methods are actually close relatives of the distributed dislocation technique for solving two-dimensional crack problems, which emerged from pioneering works by Eshelby et al. (1951), Louat (1962), Bilby et al. (1963), Keer and Mura (1966), Bilby and Eshelby (1968), Erdogan (1969), Comninou (1977) and Marcinkowski (1979). In this method, dislocations rather than dislocation dipoles are distributed along the crack, with the unknown dislocation density now corresponding to the displacement jump gradient. Solving crack problems with the distributed dislocation method involves approximating the density function by a series of orthogonal polynomials multiplied by a square root singular weight at the crack tips (Hills et al., 1996). Provided that the dislocation solutions are available for the geometry under consideration, this technique provides highly accurate estimation of the stress intensity factor. However, it is not as flexible for modeling propagating cracks, especially curved ones, as the DDM. Indeed, crack extension in the DDM is directly simulated by adding a new element to the crack tip. This particular feature of the DDM makes it also attractive in comparison to the finite element methods for problems, where the medium can be assumed to be elastic and homogeneous.

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