International Journal of Solids and Structures 48 (2011) 2764–2777

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

# Three kinematic representations for modeling of highly flexible beams and their applications

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ARTICLE INFO

*Article history:*
Received 3 February 2010
Received in revised form 28 April 2011
Available online 12 June 2011

*Keywords:*
Geometrically exact beam theories
Highly flexible beams and cables
Jaumann strains
Nonlinear structural mechanics

ABSTRACT

Presented here are three kinematic representations of large rotations for accurate modeling of highly flexible beam-like structures undergoing arbitrarily large three-dimensional elastic deformation and/or rigid-body motion. Different methods of modeling torsional deformation result in different beam theories with different mathematical characteristics. Each of these three geometrically exact beam theories fully accounts for geometric nonlinearities and initial curvatures by using Jaumann strains, exact coordinate transformations, and orthogonal virtual rotations. The derivations are presented in detail, a finite element formulation is included, fully nonlinear governing equations and boundary conditions are presented, and the corresponding form for numerically exact analysis using multiple shooting methods is also derived. These theories are compared in terms of their appropriate application areas, possible singular problems, and easiness for use in modeling and analysis of multibody systems. Nonlinear finite element analysis of a rotating beam and nonlinear multiple shooting analysis of a torsional bar are performed to demonstrate the capability and accuracy of these beam theories.



## 1. Introduction

Highly flexible beams are used in many mechanical, civil, aerospace, and architectural systems (Gantes, 2001; Pai, 2007), such as helicopter rotor blades, wings of high-altitude long-endurance aircrafts, aviation propeller blades, wind-turbine blades, robot manipulators, slender space structures for buildings, arm-type positioning mechanisms of magnetic disk drives, and flexible links for high-speed slider-crank mechanisms. Moreover, NASA's various science missions have extensively used deployable/inflatable structures consisting of highly flexible beams in order to reduce the stowed volume and weight during launch, minimize extra vehicular activities in space, and/or reduce the operation time and cost (Pai, 2007; Jenkins, 2001). Furthermore, although cables are beams with very small bending rigidity, study of loops and kinks of cables requires the use of a nonlinear beam model that can account for dramatic geometric nonlinearities.

Today's surgical and medical treatments for many diseases depend on the use of highly flexible beam- or cable-like medical devices and tools. For example, natural orifice translumenal endoscopic surgery (NOTES) has recently emerged as a favorable surgical technology under worldwide research and development (Sporn et al., 2008; Miedema et al., 2008). NOTES uses flexible endoscopic devices to access body cavities through the mouth, vagina, or anus without skin incisions and perform common surgical procedures and retraction. Such surgical tools are required to be small in diameter and highly flexible in order to follow and pass through the gullet, intestine, natural orifices and/or endoscope channels easily and safely but do not undergo kinking, permanent deformation, or breaking during surgery.

Even in molecular biology, nonlinear beam theories are needed (Yang et al., 1993; Shi and Hearst, 1994; Schlick, 1995). The supercoiling (or writhing) of DNA is known to affect every physical, chemical, and biological property of a molecule. Because supercoiled DNA is an important functional state active in the processes of replication, transcription, and recombination, it is important to model and predict structural properties of DNA in higher-order forms (supercoils, knots, catenanes, and protein-DNA complexes) in order to understand DNA's fundamental functions, including strand unwinding (replication, transcription) and passage (knotting and catenation), looping, and slithering. For modeling and analysis, a double-helical DNA polymer can be modeled as a very thin beam with initial curvatures, and nonlinear buckling analysis is the main task for understanding DNA's fundamental functions. This has stimulated many theoretical, computational, and experimental studies of mechanics and stability of thin beams, including the controlled buckling of elastic beams and the ingenious manipulations of DNA strands (Schlick, 1995).

Hence, to advance theoretical structural mechanics for today's science and engineering applications, it is important to derive a geometrically exact beam theory that can be used to investigate

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doi:10.1016/j.ijsolstr.2011.06.001