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An elasto-plastic theory of dislocation and disclination fields

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ABSTRACT

A linear theory of the elasto-plasticity of crystalline solids based on a continuous representation of crystal defects – dislocations and disclinations – is presented. The model accounts for the translational and rotational aspects of lattice incompatibility, respectively associated with the presence of dislocations and disclinations. The defects content relates to the incompatible plastic strain and curvature tensors. The stress state is described by using the conjugate variables to strain and curvature, i.e., the stress and couple-stress tensors. Defect motion is described by two transport equations. A dynamic interplay between dislocations and disclinations results from a disclination-induced source term in the transport of dislocations. Thermodynamic guidance provides the driving forces conjugate to dislocation and disclination velocity in a continuous context, as well as admissible constitutive relations for the latter. When dislocation and disclination velocity vanish, the model reduces to deWit's elasto-static theory of crystal defects. It also reduces to Acharya's linear elasto-plastic theory for dislocation fields when the disclination density is ignored. The theory is intended for use in instances where rotational defects matter, such as grain boundaries. To illustrate its applicability, a finite high-angle tilt boundary is modeled using a disclination dipole and its behavior under tensile loading normal to the boundary is shown.

1. Introduction

With motivations deriving from Weingarten's theorem (Weingarten, 1901), disclinations and dislocations were simultaneously introduced by Volterra, as early as the turn of the last century (Volterra, 1907). Dislocations are the crystal defects arising from translational lattice incompatibility, as measured by the Burgers vector, whereas disclinations are defects originating in the rotational incompatibility of the crystal lattice, as characterized by the Frank's vector (deWit, 1970). Disclinations have long been considered as secondary topics in the field theory of crystal defects, due to the very large level of elastic energy they involve, as compared with dislocations, which precludes their occurrence as isolated crystalline objects (Friedel, 1964). However self-screened configurations, such as disclination dipoles, involve relatively small elastic energy levels (Romanov and Vladimirov, 1992; Romanov and Kolesnikova, 2009). Hence, they may enter the description of the lattice structure when a single-valued elastic rotation field does not exist. Grain boundaries are such instances and, as rotational defects, disclinations may prove useful in their modeling (Li, 1972). A series of examples in relation with this idea was given in a recent review paper (Kleman and Friedel, 2008) (also concerned with liquid crystals), namely: high-angle boundaries, grain boundary ledges, grain boundaries as sources and sinks for dislocations, grain rotation in polynanocrystals. . .Yet, as suggested above, dislocation-based models have been preferentially used for that purpose over the last decades. Employing the Frank-Bilby surface-dislocation concept (Frank, 1950; Bilby, 1955), they have become widely accepted for low angle boundaries. Well-known examples are the tilt and twist boundaries. However, the dislocation-based models suffer from several limitations. Considering infinite dislocation walls makes it difficult to model the three-dimensional network of grain boundaries in a polycrystal. Accounting for high-angle boundaries requires packing dislocations so tightly along the interface that their cores must overlap. Perhaps more to the point, boundaries are seen as infinitely thin planes. Yet, grain boundaries feature spatial patterns referred to as structural units spreading over a finite width area (Sutton and Vitek, 1983). Disclination-based models remove these limitations. They may be used to model finite high-angle boundaries, and to account for their fine structure (Li, 1972; Shih and Li, 1975; Gertsman et al., 1989; Hurtado et al., 1995).

In the present paper, the aim is to present a field defect (dislocation and disclination) theory for crystal plasticity accounting for both the translational and rotational aspects of lattice incompatibility. To focus on the relevant physical ideas and avoid the complications arising from geometric nonlinearity, we limit the presentation

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