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Characteristics of the process zone at sharp notch roots

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ABSTRACT

The classical Williams solution for the state of stress at the tip of a semi-infinite notch is re-visited and the two-term singular solution re-written in a form making the mode mixity and load magnitude explicit. This is used to show that, for a 270° solid angle, the majority of notch problems exhibit a process zone which is entirely or substantially mode I in character, which in turn means that the notch strength may practically be governed by a single elastic parameter. A method for finding the practical limit on the load and stress intensity ratio where this holds is described.

1. Introduction

The solution for state of stress at the apex of a sharp notch has been with us for about thirty years, thanks to the studies carried out in the 80's and 90's (see e.g. Costabel and Dauge, 1993; Costabel et al., 1994; Dauge, 1988; Grisvard, 1985, 1992; Leguillon and Sanchez-Palencia, 1987; Kondrat'ev and Oleinik, 1983; Maz'ya et al., 1983, 1991), which follow the pioneering work performed more than half a century ago by Williams (Williams, 1952), and after whom the solution is, probably generously, often named. It has proved useful in characterising the elastic stress state as a hinterland to a plastic process zone, and has been extensively used in this context (see e.g. Atzori et al., 2005, 2009; Barber, 2010; Carpinteri et al., 2006, 2008, 2009, 2010; Dini and Hills, 2004a,b, 2006; Harding et al., 2010; Lazzarin and Filippi, 2006; Leguillon, 2001, 2002; Sinclair, 2004a,b; Susmel and Taylor, 2003). There have also been numerous extensions to the basic eigensolution itself, including determining the characteristics of bonded elastically dissimilar wedges (Bogy, 1971a; Bogy and Wang, 1971b) and elastic anisotropy (Ma and Hour, 1989, 1990; Labossiere and Dunn, 1998, 1999; Paggi and Carpinteri, 2008).

Although the elastic analysis has advanced a long way, the practical use of notch-tip intensity factors as characterizes of crack nucleation has been less popular (Cornetti et al., 2006, 2010; Leguillon, 2002), partly, perhaps, because sharp features are normally avoided in engineering design, but partly, also, because the characteristic singular solution is not 'self-similar'. This puts the general solution (for an arbitrary notch angle), at a disadvantage compared with the more familiar special case of a 360° notch (a crack), where both singular eigensolutions have the same strength (square root singular), and so the crack-tip stress field does not vary with radial coordinate within the domain in which the singular solution, as a whole, dominates the behaviour. We contrast this with all other cases, and note the temptation to assume that the first (dominant) eigensolution is appropriate as a quantity to correlate strength. The analysis reported here was partly prompted by an experimental investigation by Skallerud and co-workers (Shang et al., 2008, 2009a,b) in which this assumption was made, and provides a rigorous answer to the question of its appropriateness.

The problem will be tackled in two parts; first we look at the semi-infinite wedge solution itself, and make deductions about its properties. We will then paste the eigensolution into some simple finite problems and make further inferences about the properties of the process zone. The entire calculation is carried out using a 'plane' elasticity solution and, in the application of this solution, this will normally be interpreted as 'plane strain'. Thus, the inferences made will be limited to cases where plane strain obtains in the practical problem: in this context, it should be noted that, in the case of a straight notch through the thickness of a flat sheet, this assumption will be restricted to the central section of the plate, well away from the free faces. Recently, considerable progress has been made in analyzing problems of this class (see, e.g. Berto et al., 2011a,b; and the review of the problem by Kotousov, 2010). The last paper shows that the effects of the free surface may penetrate relatively deeply inwards from the free surface, particularly if the material displays a strong Poisson effect.

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