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Single beam analysis of damaged beams verified using a strain energy based damage measure

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ABSTRACT

Analytical expression of a new damage measure which relates the strain energy, to the damage location and magnitude, is presented in this paper. The strain energy expression is calculated using modes and natural frequencies of damaged beams that are derived based on single beam analysis considering both decrease in mass and stiffness. Decrease in mass and stiffness are a fallout of geometric discontinuity and no assumptions regarding the physical behavior of damage are made. The method is applicable to beams, with notch like non-propagating cracks, with arbitrary boundary conditions. The analytical expressions derived for mode shapes, curvature shapes, natural frequencies and an improved strain energy based damage measure, are verified using experiments. The improvement in the damage measure is that it is not assumed that the bending stiffness of the damaged beam is constant, and, equal to that of undamaged beam when calculating the strain energy of the entire beam. It is also not assumed that the bending stiffness of the element in which the damage is located is constant.

1. Introduction

Most of the literature in the area of Structural Health Monitoring (SHM), has been directed at developing methods to predict the damage location and the damage magnitude using dynamics based experimental measurements. There are some mathematical models, that give analytical theory to model the damage. Such mathematical models for structures with damage are useful in two ways, firstly; they allow understanding of the physics behind the problem, which helps in the explanation of experimental readings. Secondly, they allow prediction of response of the structure. These studies are also useful for the development of new experimental techniques. At a broad level, overviews of the approaches used for SHM can be found in Staszewski et al. (2004) and Doebling et al. (1998).

1.1. Mathematical models

Krawczuk (2002) used concepts of fracture mechanics to develop a mathematical model for cracked beams. Knowledge of K_I, K_{II} and K_{III} is required to apply the theory presented. Approximate methods were used by Christides and Barr (1984) who used the Rayleigh–Ritz method, Shen and Pierre (1990) who used the Galerkin Method, and Qian et al. (1991) who used Finite Element model to predict the behavior of a beam with an edge crack. Law and Lu (2005) used assumed modes and modeled the crack mathematically as a Dirac delta function. Wang and Qiao (2007) approximated the modal displacements using Heaviside's function which meant that modal displacements were discontinuous at the crack location.

Other ways in which the rectangular edge defect has been modeled are as a spring (Ismail et al., 1990), an elastic hinge (Ostachowicz and Krawczuk, 1991), a cut-out slot (Kirshmer, 1944) and a pair of concentrated moment couple (Thompson, 1949). It has also been modeled by Joshi as a zone with reduced Young's modulus (Joshi and Madhusudhan, 1991) and as Bilinear stiffness (Ballo, 1999). Crack function models were used by Chondros et al. (1998), while breathing cracks concept was used by Cheng et al. (1999) to model cracks.

Luo and Hanagud (1997) uniquely formulate an integral equation to model a structure with notch type damage, and, successfully demonstrate a solution technique using perturbation method. Their mathematical treatment of the damaged structures offers a theoretical means to perform the parametric studies on the damage location and size. Next Luo and Hanagud (1998) and subsequently Lestari (2001) have proposed a perturbation method to describe the behavior of damaged beams. Lestari has used Fourier sine series expansion for the modes of damaged beams with simply supported ends. While Sharma et al. (2005, 2008) "assumed the approximate solution" of the damaged plate in terms of the double Fourier sine series again for a plate simply supported on all four ends. However, both Lestari and Sharma did not obtain the complementary solution, but only the particular integral for the higher order equations.

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