



# An analytical model on the steady-state deformation of circular tubes under an axial cutting deformation mode

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## ABSTRACT

An analytical model for the steady-state axial cutting of circular tubes by a cutter with multiple blunt blades and without/with the presence of a curved surface profile deflector was developed. Experimental observations have indicated that there exist six different dissipation mechanisms. These include the outward bending of cut sidewalls, the formation of cylindrical flaps in the transient and stable cutting regions, circumferential membrane stretching, chip formation and friction. In the analytical model development, rigid-perfectly plastic material behaviour obeying the von Mises yield criterion was employed to determine the steady-state stresses and fully plastic bending moments for each deformation zone which were assumed to be uncoupled to each other. The effect of the friction force was included to the proposed solution and the total axial cutting force was determined through use of the principle of virtual power. The proposed analytical model was validated by comparing the predicted cutting force to experimental data and the effects of tube wall thickness, number of cutter blades, and extrusion diameter were investigated. A good correlation was found between the theoretical predictions and experimental observations.

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## 1. Introduction

Energy dissipation systems are required to absorb the impact energy in accidents at a constant force with a long stroke over the total available stroke. In addition, energy dissipation systems are also required to exhibit good controllability and repeatability over a range of impact velocities. Axial splitting of square and circular tubes has been recognized as an efficient energy dissipation mechanism for obtaining a relatively constant force over a long stroke (Stronge et al., 1983; Reddy and Reid, 1986; Huang et al., 2002a,b). Axial cutting of circular tubes by a cutter with multiple blunt cutting blades is another effective energy absorption deformation mode with a long stroke efficiency (Jin et al., 2006). Circular AA6061-T6 extrusions underwent axial cutting and exhibited high crush force efficiencies up to 95% and a favourable constant steady-state cutting force. The load/displacement and energy absorption characteristics of circular AA6061-T6 extrusions under axial cutting were further studied by Jin et al. (2008) and Jin and Altenhof (2010) with regards to the effects of tube wall thickness and number of cutter blades. An increasing relationship was observed between the steady-state cutting force and the tube wall thickness as well as between the steady-state cutting force and the number of cutter blades. In an effort to reduce the spatial requirement of the cutting system and to ease the flaring of cut petalled sidewalls,

a cone-shape deflector with a straight or curved profile was designed by Jin and Altenhof (2008) and implemented into the cutting test apparatus. Minor influence on the load/displacement and energy absorption characteristics were observed for the axial cutting tests with the presence of the deflector. Five energy dissipation mechanisms were identified with and without the use of the deflector, namely, cutting deformation, circumferential membrane stretching of the extrusion, petalled sidewall outward bending, far-field sidewall material fracture, and friction between the cutter blade and the extrusion sidewall.

The problem of wedge cutting a thin-wall plain plate has similar energy dissipation mechanisms as the problem of axial cutting of a circular tube by a cutter with multiple blades. The deformation process of a wedge cutting a plate, as one of the primary energy absorbing mechanisms, has received considerable attention and a thorough literature review dealing with experimental and theoretical analyses of the plate cutting resistance force by a sharp wedge was presented by Lu and Calladine (1990) and Simonsen and Wierzbicki (1998).

Although the mechanics of the cutting process is complicated, the analysis of the cutting process falls into two stages: initial blade/wedge penetration (transient cutting stage) and steady-state cutting. The transient stage considers first contact between the blade/wedge tip and tube/plate edge to the state where the resistance force reaches a constant level. If the blade/wedge has a finite width, the cutting resistance force will reach a constant value after a certain penetration depth and the process is then said to be

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