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# Elastic spherical shell impacted with an elastic barrier: A closed form solution

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ABSTRACT

In this paper, the impact of a thin-walled elastic spherical shell with an elastic barrier is investigated. Hertzian and Reissner (membrane–bending deformation) theories were employed for the deformation equations. Due to the complexity of the equations a linearization of the equations was proposed and a closed form solution of the problem was obtained. Newtonian method is applied in order to obtain the impact force and the time duration. The closed-form solution enables one to parametrically study the impact and the related quantities. Finally the results from the analytical solution are validated by the finite element method and also are compared with the results presented in Young (2003). The comparison of the results reveals a good agreement. It is concluded that the proposed closed-form solution can be used to parametrically assess the impact of elastic spherical shells to elastic half space.


## 1. Introduction

Contact problems of two deformable elastic bodies have been studied for decades. These problems become more complex when the bodies are impacted to one another, due to the transient and nonlinear nature of the problem. In particular, impact analysis of spherical shells is important and has attracted investigator's attention due to their wide range of applications in solid mechanics and biomechanics.

The impact loading of spherical shells has been the subject of many theoretical and experimental studies in engineering. Many attempts have been made to employ spherical shell theory and to perform impact analyses. For example, Reissner (1947) obtained explicit results for shells with and without edge restraints carrying either point or distributed loads. He concluded that center deflection is a function of shell radius '$h$' to shell thickness '$t$'. Furthermore Reissner (1959) investigated the solution to some problems using membrane theory and by solving the equation of flexure for cantilever spherical shell. Engin (1969) proposed an analytical model which included bending and membrane compliance of the shell and evaluated the response for a fluid filled shell subjected to delta-function type loading. In addition the cases of a hollow spherical shell were also considered by Engin. In the experimental studies of Kenner and Goldsmith (1972), they obtained the magnitude of the impact force and the strains at different locations for both fluid filled and hollow spherical shells. Kunukkasseril and Palaninathan (1975) conducted impact experiments on shallow spherical shells to produce different pulse durations, they measured the impact force and strains. Hammel (1976) developed and solved a linear integral equation for an unknown impact force. He considered a particle striking a spherical shell, and concluded that the elastic deformation of the shell was much less than the deformations of a plate of equal thickness due to the same impact load. Senitskii (1982) developed Hammel's approach further by including the effect of local deformations in the shell due to Hertzian stress field. Stein and Wriggers (1982) studied impact-contact problems of thin elastic shells taking into account geometrical nonlinearities within the contact region.

Numerical results were also obtained for the impact-contact problem of spherical shells. Koller and Busenhart (1986) investigated elastic impact of spheres on thin spherical shells. A nonlinear integral–differential equation of the impact process was developed on the basis of Reissner's approximate theory and the quasi-static Hertzian contact theory. Dynamic buckling of a thin shallow spherical shell under impact loads was numerically calculated by Chun et al. (1992). They concluded that the critical buckling load increased with the enlargement of the loading area. Sabodash and Zhemkova (1993) investigated the dynamic reaction of a spherical shell which was subjected to the local effect of a normal pressure pulse. A numerical method based on the characteristic relationships was developed. Trial calculations were performed, and the numerical results were analyzed. Consequently a number of mechanical effects of practical and scientific interest were established by Sabodash and Zhemkova (1993). Pauchard and Rica (1998) studied the deformation of a thin elastic shell striking a rigid plane, and in another case, he investigated the elastic shell subjected to a localized load based on the total elastic energy. He also determined the restitution coefficient of the shell during impact, in his research

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