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Analysis of bi-material interface cracks with complex weighting functions and non-standard quadrature

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ABSTRACT

A boundary element formulation is developed to determine the complex stress intensity factors associated with cracks on the interface between dissimilar materials. This represents an extension of the methodology developed previously by the authors for determination of free-edge generalized stress intensity factors on bi-material interfaces, which employs displacements and weighted tractions as primary variables. However, in the present work, the characteristic oscillating stress singularity is addressed through the introduction of complex weighting functions for both displacements and tractions, along with corresponding non-standard numerical quadrature formulas. As a result, this boundary-only approach provides extremely accurate mesh-insensitive solutions for a range of two-dimensional interface crack problems. A number of computational examples are considered to assess the performance of the method in comparison with analytical solutions and previous work on the subject. As a final application, the method is applied to study the scaling behavior of epoxy–metal butt joints.

1. Introduction

Bi-material composites and structures are prevalent throughout a broad range of natural and engineered systems due to the ability of such systems to take advantage of the positive attributes of the individual constituents and to minimize their weaknesses. However, the interface between the two materials often remains as a critical region, thus limiting the overall performance of the composite. For systems that can be idealized as remaining essentially in the linear elastic regime, the stress intensity factors at the tips of any interfacial cracks or flaws can be used as controlling parameters. In the present paper, we develop a new boundary element formulation to evaluate these stress intensity factors for the two-dimensional case. The problem is complicated by the fact that the stress field around the interfacial crack tip is singular and log-periodic. This, in turn, necessitates the calculation of a complex stress intensity factor in which the usual opening and shear modes are intrinsically coupled.

Significant contributions toward understanding the physical and mathematical bi-material crack problem include the work by Muskhelishvili (1953); Williams (1959); Sih and Rice (1964); England (1965); Erdogan (1965); Rice and Sih (1965); Bogy (1971); Comninou (1977); Hutchinson et al. (1987) and Rice (1988). In terms of computational methods, Lin and Mar (1976) developed the first finite element formulation to analyze a crack between dissimilar materials, while Barsoum (1974) introduced the concept of quarter-point elements that has been used successfully for a range of fracture mechanics problems. Early work on the development of the boundary element method for linear elastic fracture mechanics includes that by Cruse (1978) and Blandford et al. (1981). The bi-material crack problem has been addressed more recently within the context of a boundary element method by Lee and Choi (1988); Yuuki and Cho (1989); Raveendra and Banerjee (1991); Tan et al. (1992); Ang et al. (1996); Lee (1996); Lim et al. (2002) and Zhou et al. (2005).

In the next section, we begin by providing the governing equations, along with an overview of the basic characteristics of the response for a bi-material crack in elastic media. This is followed by a presentation of the existing quarter-point boundary element fracture mechanics formulations, including the work by Raveendra and Banerjee (1991) for the bi-material problem. This sets the stage for our current development of a boundary element approach that explicitly addresses the singular, log-periodic behavior of the solution near the crack tip. Here we first consider the formulation for an elastic material bonded to a rigid body with a planar interfacial crack and then subsequently examine the more general bi-material problem. Afterwards, the results of several numerical examples are examined in comparison with analytical solutions and previously published approaches and physical experiments. The paper then finishes with some general conclusions.

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