



## Buckling of delaminated bi-layer beam-columns

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### ABSTRACT

An improved analytical model is presented to analyze the delamination buckling of a bi-layer beam-column with a through-the-width delamination. Both the transverse shear deformation and local delamination tip deformations are taken into consideration, and two delaminated sub-layers as well as two substrates in the intact (un-delaminated) regions are modeled as individual Timoshenko beams. A deformable interface is introduced to establish the continuity condition between the two substrates in the intact regions. Consequently, a flexible joint is formed at the delamination tip, and it is different from the conventional rigid joint given in most of studies in the literature, in which the local delamination tip deformations are completely ignored. In contrast to the local delamination buckling in our previous study (Qiao et al., 2010), the present model accounts for the global deformations of the intact region in the delaminated composite beam-column, thus capable of capturing the buckling mode shape transitions from the global, to global–local coexistent, and to local buckling for asymmetric delamination as the interface delamination increases. Good agreement of the present analytical solutions with the full 2-D elastic finite element analysis demonstrates the local deformation effects around the delamination tip and verifies the accuracy of the present model. Parametric studies are conducted to investigate the effects of loading eccentricity, delaminated sub-layer thickness ratio, and interface compliance on the critical buckling load for the delaminated composite beam-column. Transitions of buckling modes from the global to local delamination buckling are also disclosed as the thickness of one sub-layer reduces from the thick sub-layer to a thin film. The developed delamination buckling solution facilitates the design analysis and optimization of laminated composite structures, and it can be used with confidence in buckling analysis of delaminated composite structures.

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### 1. Introduction

Due to manufacturing defects (e.g., imperfect curing process) or in-service accidents (e.g., low velocity impact), delamination may appear in laminated composite materials. Because of the presence of the delaminated area, the designed buckling strength of laminated structures can be reduced when subjected to compressive loading. Thus, as a major failure mode, the buckling of delaminated composite structures has been extensively studied in the literature.

Numerous studies have been attempted to model and analyze the buckling problem of delaminated beam- or plate-type composite structures. Including the bending-extension coupling, Chai et al. (1981) conducted one-dimensional buckling analysis of single delaminated composite laminate plates. Later, Chai (1982) developed one of the first analytical delamination models for homogeneous, isotropic plates using a thin-film model and extended this approach to a general bending case which includes the bending of a thick base laminate. Yin (1988) derived general formulae for

thin-film strips with mid-plane symmetric delamination and evaluated the effects of laminated structures on delamination buckling and its growth. Based on the classical plate theory, Yin and his coworkers (Yin et al., 1986; Yin, 1998) conducted thermo-mechanical buckling and post-buckling analyses of multilayered laminates with an across-the-width delamination and presented the closed form solution of delamination buckling mode.

Due to low transverse shear modulus relative to the longitudinal Young's modulus in composite laminates, shear deformation was shown as a key factor influencing the delamination buckling behavior. Based on a variational energy approach, Chen (1991) proposed a shear deformation theory to study the delamination of one-dimensional (1-D) orthotropic homogeneous elastic beams. Considering large deflection of laminates, Chen (1993) later derived the closed form expressions for the critical buckling load and post-buckling deflection of asymmetric laminates. According to the studies (Chen, 1991, 1993), the inclusion of transverse shear deformation reduces the overestimation of buckling and ultimate load capacity of delaminated composite plates. In virtue of a perturbation technique, Kardomateas and Shmueser (1998) investigated the effect of transverse shear on buckling and post-buckling of a 1-D orthotropic elastic beam with a through-width

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