



An implicit tensorial gradient plasticity model – Formulation and comparison with a scalar gradient model

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ABSTRACT

Many rate-independent models for metals utilize the gradient of effective plastic strain to capture size-dependent behavior. This enhancement, sometimes termed as “explicit” gradient formulation, requires higher-order tractions to be imposed on the evolving elasto-plastic boundary and the resulting numerical framework is complicated. An “implicit” scalar gradient model was thus developed in Peerlings [Peerlings, R.H.J., 2007. On the role of moving elastic–plastic boundaries in strain gradient plasticity. *Model. Simul. Mater. Sci. Eng.* 15, 109–120] that has only C^0 continuity requirements and its implementation is straightforward. However, both explicit and implicit scalar gradient models can be problematic when the effective plastic strains do not have smooth profiles. To address this limitation, an implicit tensorial gradient model is proposed in this paper based on the generalized micromorphic framework. It is also demonstrated that the scalar and tensorial implicit gradient models give similar results when the effective plastic strains fluctuate smoothly.

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1. Introduction

Classical constitutive models for metals are not capable of predicting a size-dependent behavior as observed experimentally at the micron level (Fleck and Hutchinson, 1997; Stölken and Evans, 1998). To capture the size effect phenomena, several higher-order continuum theories have been proposed. These models typically incorporate length-scale parameters which are associated with the gradients of plastic strain.

One class of such theories introduces a length scale parameter in the incremental tangent modulus to reflect the additional hardening at small geometrical dimensions (Bassani, 2001). This approach preserves the structure of conventional plastic theories and does not involve higher order boundary conditions. Niordson and Hutchinson (2003b) have adopted such enhancements for a simple shear problem and they question the subsequent localization behavior observed in this example. Moreover, Volokh and Hutchinson (2002) reported that the absence of higher-order boundary conditions results in non-unique solutions. However, based on uniqueness arguments, Acharya et al. (2004) showed that additional boundary conditions can be admitted in these so-called lower order gradient theories and which the localization behavior is avoided with proper numerical treatments.

Another broad class of gradient plasticity theories is inspired by the work of Aifantis (1984) where the second gradient of the plastic strain is incorporated in the yield function and which can be interpreted as a special case of the formulation by Fleck and Hutchinson (2001). We term these models that require the derivatives of the plastic strain as “explicit” gradient enhancements. The thermodynamics-based formulation for this class of models showed that the gradient terms characterize the stored energy due to the presence of defects such as dislocations and entanglements (Gurtin and Anand, 2009). Since the yield criterion is satisfied only in the plastic domain, the higher-order boundary condition associated with it has to be imposed at the evolving elasto-plastic boundary (Peerlings, 2007). For a similar explicit gradient enhanced softening model, de Borst and Pamin (1996) achieved this requirement numerically by imposing either C^1 continuity or having the first gradient of the scalar plastic strain as degrees of freedom. Alternatively, these gradient enrichments can also be applied to viscoplastic constitutive relations so that distinct elasto-plastic boundaries are avoided (e.g. Gudmundson, 2004). Rate-independent behavior can be approximated by having small values for the rate-sensitivity exponents, although numerical issues may arise as these parameters tend to zero.

In view of the numerical difficulties inherent in the explicit gradient models, an alternative class of so-called “implicit” gradient formulations with only C^0 continuity requirements was developed. These gradient enhancements are generally utilized to avoid mesh dependency issues during softening (e.g. Peerlings et al., 1996; Engelen et al., 2003; Poh and Swaddiwudhipong, 2009) and have

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