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Warping of stationary and rotating heavy disks

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ABSTRACT

A flexible circular plate with gravity as its main source of external loading is called a heavy disk. This paper studies the non-axisymmetric warping of a clamped-free heavy disk. In the theoretical formulation, von Karman's plate model is adopted to derive the equations of motion of a rotating heavy disk. In the experiment, a $5\frac{1}{4}$-inch floppy disk is used to demonstrate the non-axisymmetric deformations when the disk is either stationary or rotating. We first measured the shape of the floppy disk when it was stationary. It was observed that two non-axisymmetric deformations coexisted, one with shape $\cos 2\theta$ and the other $\cos 3\theta$. We next rotated the floppy disk. It was observed that when the floppy disk started with shape $\cos 2\theta$, the deformation switched to shape $\cos 3\theta$ as the rotation speed increased to 303 rpm. As the speed continued to increase, the shape $\cos 3\theta$ persisted all the way up to 1010 rpm. Beyond that the disk deformation became axisymmetric until 1313 rpm. Beyond 1313 rpm, aeroelastic flutter started to kick in and equilibrium state was no longer achievable. The rotation speed was then reduced gradually from high speed to 0. It was found that the deformation remained $\cos 3\theta$ all the way, instead of switching back to the $\cos 2\theta$ shape the disk started out with in the first place. The coexistence of two stable non-axisymmetric deformations of the stationary disk and the hysteresis behavior of the rotating disk observed in the experiment are consistent with theoretical predictions.

1. Introduction

In most of the researches in circular plate deformation, stationary or rotating, gravity is often neglected although it is ubiquitous on earth. This ignorance is justified in some applications, especially when the deflection due to the disk's own weight is small compared to other working loads. Another possible reason for the lack of interest in this issue is because it appears to be a trivial problem. It is intuitive to envisage that the disk deforms axisymmetrically like a mushroom under its own weight, see Wang (2004).

In some other applications this might not be the case, especially when the disk is thin and flexible. This can occur in floppy disk drives. For Iomega zip disk with 3.5-inch diameter the maximum edge deflection of the disk under its own weight can reach 20 times of the disk thickness. The warping is even more serious in the older model of Bernoulli disk with $5\frac{1}{4}$-inch diameter. These flexible disks deform non-axisymmetrically when they are not rotating.

A flexible circular plate with its own weight as the main source of external loading is called a "heavy" disk in this paper. Recently, Chen and Fang (2010) used a paper disk to demonstrate that a very flexible disk would droop non-axisymmetrically instead of axisymmetrically. They used von Karman's plate model to formulate a theory which predicts that a heavy circular plate resting on a ring of elastic foundation may deform in a non-axisymmetric manner when the external uniform load reaches a critical value. The results reported by Chen and Fang cannot be obtained if Kirchhoff's plate model is adopted (Wang, 2004). Although Chen and Fang also predicted theoretically that multiple stable non-axisymmetric deformations may coexist under certain conditions, no experimental proof was provided to support this prediction.

In this paper we demonstrate, both theoretically and experimentally, the coexistence of more than one stable non-axisymmetric configuration when a clamped-free disk deforms under its own weight. von Karman's plate model is adopted to derive the equations of motion of a rotating heavy disk. A nonlinear Galerkin's method based on two sets of assumed functions is used to solve the equilibrium equations. A vibration method is used to determine the stability of the calculated deformations. For a stationary disk, we are interested in the variation of the disk deformation and its natural frequencies as the external force (gravity) varies. For a rotating disk, we will focus on how the stable non-axisymmetric deformations evolve as the rotation speed changes. Experiments on a $5\frac{1}{4}$-inch floppy disk are conducted to examine the theoretical predictions on the behaviours of the stationary and rotating disks.

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