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Micro-cracks informed damage models for brittle solids

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ABSTRACT

A class of micro-cracks informed damage models for describing the softening behavior of brittle solids is proposed, in which damage evolution is treated as a consequence of micro-crack propagation. The homogenized stress–strain relation in the cracked microscopic cell defines the degradation tensor, which can be obtained by the equivalence between the averaged strain energy of the microscopic cell and the strain energy density of the homogenized material. This energy equivalence relationship serves as an energy bridging vehicle between the damaged continuum and the cracked microstructure. Several damage evolution equations are obtained by this energy bridging method. The size effect of the micro-cracks informed damage law is characterized through the microscopic cell analysis, and the proper scaling of the characterized damage evolution functions to eliminate mesh dependency in the continuum solution is introduced.

1. Introduction

Continuum damage mechanics (CDM) has been an active research area for more than 50 years (Kachanov, 1958; Robotnov, 1968; Lemaitre and Chaboche, 1974; Mazars, 1984; Ju, 1989; Faria et al., 1998). However, CDM is generally phenomenological, where the damage evolution functions are not related to specific microstructural parameters and local defects. Some earlier efforts have been devoted to the micromechanical investigation of cracked solids (Budiansky and O'connell, 1976; Norris, 1985; Hashin, 1988), however the link to CDM is lacking.

Damage evolution based on a multi-scale framework has been investigated in recent years. Lee et al. (1999) combined the standard finite element method (FEM) with asymptotic homogenization in the undamaged domain and the voronoi cell finite element model (VCFEM) in the damaged region. Fish et al. (1999) proposed a nonlocal damage theory based on asymptotic homogenization to account for damage effects in a heterogeneous media. The reliability of this approach, however, depends on reliability of the micro-scale damage models. Dascalu et al. (2008) and Dascalu (2009) defined the damage variable directly by the micro-crack length. Then a series of differential equations was introduced to describe damage evolution in the microscopic cell based on the homogenization theory and the energy controlled crack propagation. By introducing an internal length in the damage evolution equation, the size effect was characterized.

Damage processes in brittle solids are driven by the distribution of micro-cracks and their evolution. Explicit modeling of micro-cracks in brittle solids is computationally infeasible, while classical damage models are phenomenological in nature; missing the link to microscopic properties. Micromechanics based approaches have been introduced (Nemat-Nasser and Hori, 1999) to obtain homogenized material properties of cracked solids. However, they are limited to certain idealized crack configurations and loading conditions. Alternatively, asymptotic expansion offers a rigorous means for relating physical variables defined at different scales (Bensoussan et al., 1978; Bakhvalov and Panasenko, 1989; Guedes and Kikuchi, 1990; Fish et al., 1997). However, the key step in an asymptotic type method is solution of the characteristic functions in the microscopic cell, which is typically very time consuming.

The present work aims at constructing damage models based on thermodynamics of "cracked" microscopic cells and the corresponding "damaged" macroscopic continua. Crack evolution in the microscopic cell is first modeled numerically. Then, corresponding damage evolution functions in the continuum are constructed through a Helmholtz free energy relationship between damaged and undamaged homogenized continua. This approach avoids the tedious solution of characteristic functions in the conventional asymptotic type method.

An outline of this paper is as follows. In Section 2, the two-scale model problem is stated and the homogenization procedures are defined. Based on the homogenized stress and strain, the energy relation between micro- and macro-scales is established in Section 3. In Section 4, procedures for constructing damage evolution functions based on the cracked microscopic cell solution for a

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