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On the limit velocity and buckling phenomena of axially moving orthotropic membranes and plates

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ABSTRACT

In this paper, we consider the static stability problems of axially moving orthotropic membranes and plates. The study is motivated by paper production processes, as paper has a fiber structure which can be described as orthotropic on the macroscopic level. The moving web is modeled as an axially moving orthotropic plate. The original dynamic plate problem is reduced to a two-dimensional spectral problem for static stability analysis, and solved using analytical techniques. As a result, the minimal eigenvalue and the corresponding buckling mode are found. It is observed that the buckling mode has a shape localized in the regions close to the free boundaries. The localization effect is demonstrated with the help of numerical examples. It is seen that the in-plane shear modulus affects the strength of this phenomenon. The behavior of the solution is investigated analytically. It is shown that the eigenvalues of the cross-sectional spectral problem are nonnegative. The analytical approach allows for a fast solver, which can then be used for applications such as statistical uncertainty and sensitivity analysis, real-time parameter space exploration, and finding optimal values for design parameters.

1. Introduction

As is well known, the mechanical behavior of paper under a non-failure condition is adequately described by the model of an elastic orthotropic plate. The rigidity coefficients of the plate model, describing the tension and bending of the paper sheet, have been estimated for various types of paper in many publications (see, for example, Göttsching and Baumgarten, 1976; Thorpe, 1981; Skowronski and Robertson, 1985; Seo, 1999). The deformation properties of a sheet of paper under tensile stress or strain are used in simulation of axial movement of a paper web. In particular, these properties are important for modeling the instability of the axially moving elastic web.

There are numerous studies on the loss of stability of moving elastic webs based on one-dimensional models, using second- and fourth-order differential equations. These studies are devoted to various aspects of free and forced vibrations, including the nature of wave propagation in moving media, and the effects of axial motion on the frequency spectrum and eigenfunctions. The studies include e.g., Archibald and Emslie (1958); Miranker (1960); Swope and Ames (1963); Mote (1968, 1972, 1975); Simpson (1973); Ulsoy and Mote (1980, 1982); Chonan (1986), and Wickert and Mote (1990).

Two-dimensional studies have also been performed. Lin and Mote (1995) studied an axially moving membrane in a 2D formulation, predicting the equilibrium displacement and stress distributions under transverse loading. Later, the same authors predicted the wrinkling instability and the corresponding wrinkled shape of a web with small flexural stiffness (Lin and Mote, 1996). The stability and vibration characteristics of an axially moving plate were investigated by Lin (1997). The loss of stability was studied with application of dynamic and static approaches and the approach by Wickert and Mote (1990) to derive the equation of motion for the plate in matrix form and to use the Galerkin method. It was shown by means of numerical analysis that, for all cases dynamic instability (flutter) is realized when the frequency is zero and the critical velocity coincides with the corresponding velocity obtained from static analysis. In Shin et al. (2005), the out-of-plane vibration of an axially moving membrane was studied. Also here, it was found by numerical analysis, that for a membrane with a no-friction boundary condition in the lateral direction along the rollers, the membrane remains dynamically stable until the critical speed, at which statical instability occurs.

The two-dimensional problem of instability analysis of an axially moving elastic plate was formulated and investigated analytically in Banichuk et al. (2010) in the context of the isotropic model. It was observed that the transverse deflection localizes near the free edges. This was noted to correspond to the eigenfunctions of

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