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Elastic fields generated by inhomogeneities: Far-field asymptotics, its shape dependence and relation to the effective elastic properties

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ABSTRACT

We discuss connections between the effective elastic properties of a solid with inhomogeneities and the far-field asymptotics of the elastic fields generated by them. We focus on the dependence of the far-field asymptotics on the inhomogeneity shape. This shape dependence in the *inhomogeneity* problem is in contrast with shape *independence* of the far field in the *eigenstrain* problem. For the latter, the far field applies to inclusions of any shape. We show that the external fields in the eigenstrain – and the inhomogeneity problems are interrelated by the same tensor that characterizes the compliance contribution of an inhomogeneity.

1. Introduction

In the problem of effective elastic properties of solids with inhomogeneities, the key quantity is the compliance contribution of an inhomogeneity that gives the extra strain due to its presence. We discuss the connection between this quantity and the far-field asymptotics of the elastic fields generated by the inhomogeneity.

This connection has received some attention in literature. We mention the work of Jasiuk et al. (1994) and Jasiuk (1995) on 2-D polygonal holes. In this setting, they made an observation that the far-field asymptotics of the hole-generated fields fully determines the compliance contribution of the hole. Actually, sufficiency of the far-field extends to the general 3-D case. Indeed, for certain volume V containing an inhomogeneity, the extra overall strain due to its presence is given by the well-known expression in terms of an integral over the boundary ∂V (see, for example, the footnote remark of Hill (1963)):

$$\Delta\varepsilon_{ij} = \frac{1}{2V}\int_{\partial V}(\Delta u_i n_j + \Delta u_j n_i)dS \tag{1.1}$$

where Δu_k are extra displacements due to the inhomogeneity and n_i is the outward unit normal to ∂V. Volume V can be arbitrarily large, hence the far-field asymptotics of Δu_k (more precisely, Δu_k as functions of the applied stresses σ^0_{ij}) is sufficient for determination of the compliance contribution of an inhomogeneity.

Note that the possibility to express the said contribution in terms of the far-field asymptotics does not seem to offer any *computational* advantages since accurate calculation of the far-field by the finite element method still requires fine meshing in close vicinity of the inhomogeneity (Babuska and Strouboulis, 2001). The importance of relation (1.1) is mostly conceptual. Besides, it gives the compliance contribution of an inhomogeneity in terms of experimentally measurable quantities – displacements of the specimen boundaries; in this context, volume V *must* be large (in order to neglect the inhomogeneity-boundary interaction) thus making the far-field asymptotics *necessary*.

The far-field asymptotics is *shape-dependent*, even in cases when the inhomogeneity compliance contribution is isotropic (for example, when the inhomogeneity shape has the symmetry of any equilateral polygon, except square). In this regard, we note inconsistent statements made in literature: for example, the observation on sufficiency of far-fields made by Jasiuk et al. (1994) was followed by an incorrect statement on their shape independence ("The essential idea is that the hole is replaced by an *equivalent circular hole* for purposes of obtaining the far field and hence the effective moduli", page S21 of the mentioned work). Note that their own formulae (26–33) contradict this statement: they contain *two* shape-dependent constants, c and d; hence the radius of the presumably equivalent circular hole cannot be adjusted in such a way as to match *both* of them. Indeed, the ratio of the bulk to shear compliances of a hole is shape dependent; hence matching by an equivalent circular hole (for which the said ratio has certain specific value) is generally impossible.

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