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Random homogenization analysis in linear elasticity based on analytical bounds and estimates

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ABSTRACT

In this work, random homogenization analysis of heterogeneous materials is addressed in the context of elasticity, where the randomness and correlation of components' properties are fully considered and random effective properties together with their correlation for the two-phase heterogeneous material are then sought. Based on the analytical results of homogenization in linear elasticity, when the randomness of bulk and shear moduli, the volume fraction of each constituent material and correlation among random variables are considered simultaneously, formulas of random mean values and mean square deviations of analytical bounds and estimates are derived from Random Factor Method. Results from the Random Factor Method and the Monte-Carlo Method are compared with each other through numerical examples, and impacts of randomness and correlation of random variables on the random homogenization results are inspected by two methods. Moreover, the correlation coefficients of random effective properties are obtained by the Monte-Carlo Method. The Random Factor Method is found to deliver rapid results with comparable accuracy to the Monte-Carlo approach.

1. Introduction

The homogenization method has been developed and extended to reduce the number of composite design parameters significantly by the introduction of effective characteristics using potential or complementary energy principles (Markovic and Ibrahimbegovic, 2006; Aboudi, 1991; Zohdi and Wriggers, 2005). The method relies on a statistically representative sample of material, referred to as a representative volume element (RVE). It is a finite sized sample from the heterogeneous material that characterizes its macroscopic behavior (Aboudi, 1991; Zohdi and Wriggers, 2005; Torquato, 2002). Although this technique, in its modern version, is more than 40 years old, there are many novel approaches and applications, such as in the food industry (Kanit, 2006), some composites made of wood (Lux, 2006), superconductors (Kaminski, 2005), even for time-dependent cases by "equation free" approach (Samaey et al., 2006); a variety of materially nonlinear multi-component composites can be homogenized as well (Idiart, 2006). Following numerous engineering applications, the strength of composites can also be estimated by the homogenization method (Steeves and Fleck, 2006).

Homogenization techniques deliver effective properties of heterogeneous materials. Exact computational approaches are summarized in Zohdi and Wriggers (2005). Here, the attention is focused to estimates and bounds. In this context, early approximations for the effective properties were first developed by Voigt (1889) and Reuss (1929). In 1957, Eshelby (1957) obtained a relatively compact solution that has been a basis for many approximation methods. Based on variational principles, Hashin and Shtrikman (1962) developed a model that improved solutions of the effective properties. Additional classical models have been proposed to estimate the effective properties, including the Self-Consistent method, the dilute distribution method, and the Mori and Tanaka (1973) method. Further approaches for estimating or bounding the effective responses of heterogeneous materials can be found for instance in Aboudi (1991), Mura (1987) and Nemat-Nasser and Hori (1999).

In recent years, a lot of attention is paid to random composites because of an uncertainty in reinforcement location/shape and/or pore spatial distribution in matrices, and randomness in components. Kaminski reported the perturbation-based homogenization analysis of two-phase composites (Kaminski and Kleiber, 2000) and the perturbation-based homogenization analysis for thermal conductivity of unidirectional fiber reinforced composites (Kaminski, 2001). Sakata obtained a macroscopic response by applying stochastic homogenization analysis for unidirectional fiber reinforced composites using the Monte-Carlo simulation (Sakata

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