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Influence of a finite strain on vibration of a bounded Timoshenko beam

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ABSTRACT

The goal of this work is to study the eigenmodes of shearable beams with initial finite strain. A three dimensional model is developed on the base of Cosserat continuum mechanics. The characteristics of waves propagation superimposed upon finite pre-stress are obtained using the (rigorous) calculation of the Hamiltonian action. The results are applied on vibration of beam supporting a finite longitudinal strain. Nonlinear effect according to the pre-stress is obtained for various boundary conditions and through a nondimensional formalism.

1. Introduction

Beam models are usually applied in the broad area of mechanical or civil engineering and particularly in the general framework of structure (plate or beam) resonance technique. This method is efficient for determining the mechanical properties of new and complex materials analytically *e.g.* (Puchegger et al., 2005; Villain et al., 2009) or through numerical methods *e.g.* (Etcheverry and Sánchez, 2009; Taylor et al., 2002). The most utilized beam model remains the Euler–Bernoulli model that neglects the transversal shearing of the beam section *e.g.* (Han et al., 1999). It is suitable for most engineering applications but is rather limited to relatively low frequencies and thin beam.

In the last decade, new interest is observed for the un-stressed Timoshenko beam model since it permits to estimate beam spectrum in the domain of relatively high frequencies and for thick beam *e.g.* (Stephen, 2006; Bhaskar, 2009; Ekwaro-Osire et al., 2001). One of the reason is that nano beams have attracted attention of scientific community due to their wide applications in modern technology.

Compared to the original Euler–Bernoulli model, the Timoshenko beam adds the transversal shearing and also the rotational inertia of the beam section during transversal and flexional vibration. It induces some controversial problems about the existence of so-called double spectrum of Timoshenko beam. From experimental point of view, it was however shown that only one set of frequencies exist although it gives two standing waves *e.g.* (Chan et al., 2002). In any case, the Timoshenko beam model provides a more accurate description for motions of thick beams at high frequencies *e.g.* (Yoon et al., 2004), or vibrating due to thermal excitation *e.g.* (Hsu et al., 2008). Practically, another problem appears since engineering application often faces the presence of beam with some initial pre-stress due to fixations *e.g.* (Kang and Tan, 1998). Multibody dynamics combined with numerical method was used to analyze the nonlinear response of pre-compressed beam *e.g.* (Kovacs, 1996), but limited to low frequencies.

Higher frequency modal analysis requires the exact theoretical development of beam finite strain developed by *e.g.* (Simo, 1985). Since then, as far as we know, most of the analysis of beam finite strain is essentially numerically oriented (Auciello and Ercolano, 2004). The analysis of superimposed waves motion upon beam finite strain is far from achieved. The missing link remains the analytical determination of the continuum tangent operator. This analytic expression is necessary for calculating modal mass and modal stiffness. Indeed, in order to consider prestress effects when performing a free vibration analysis, the free vibration equation (undeformed structure) has to be modified to include the prestress effect in the stiffness matrix. Thus the second variation of the Hamiltonian action is a convenient method to obtain the corresponding stiffness operator in the eigenvalue analysis around an equilibrium point. By the way, the theory of incremental deformations around a large deformation-stressed state is usually referred to, in the nonlinear elasticity theory, as small-on-large, or incremental theory (Ogden, 1997).

The goals of the present study are first to determine the continuum tangent operator in dynamics and second to investigate the influence of the pre-stress on vibrations superimposed on an arbitrary initial finite strain for a three-dimensional Timoshenko beam. The model is applied to the in-plane vibrations of a beam supporting a longitudinal finite pre-strain.

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