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On the evolution of intrinsic curvature in rod-based models of growth in long slender plant stems

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ABSTRACT

In many applications of rod theories as models for plant stem growth and development, it is necessary to allow the intrinsic curvature and flexural stiffness of the rod to evolve. In the present paper, the application of evolution equations for these quantities is examined and a new evolution equation for the intrinsic curvature is proposed. To illustrate the new evolution equation, several examples of the evolution of rods in the presence of external forces and tip growth are presented. Growth in plants frequently results in residual stresses and the compatibility of rod models with these fields is also discussed.

1. Introduction

The growth and evolution of a plant stem features several mechanical phenomena. First, as the tip grows and the stem elongates, the added weight causes the plant to become increasingly deformed. This deformation is controlled by the lateral accretion of mass on the outer surface of the stem, a possible stiffening of the stem, and the development of intrinsic curvature. These features are evident when one cuts a stem from a rose plant and notices that the shape of the stem does not change significantly after it has been cut. They can also be observed in the oscillations of plant stems which are subject to transient loading caused by wind and rain.

The use of rod theories to model plant stems has a long history which dates at least to the seminal work by Greenhill (1881) on the stability of a column. More recent works, such as those by Silk and Erickson (1980) and Silk et al. (1982) have used Euler's theory of the elastica to model the evolution of a rice panicle, Goriely and Neukirch (2006) and McMillen and Goriely (2002) have used Kirchhoff–Love rod theory to examine tendril perversion and climbing in vines of morning glories, while Yamamoto et al. (2002) have used rod theories to examine the growth of wooded plant stems. Arguably among the most ambitious works in this area are Costes et al. (2008), Fourcaud and Lac (2003) and Fourcaud et al. (2003) who model individual branches using rods and then combine the rods to form trees.

The contribution of the present paper to the literature is to present a novel evolution equation for the intrinsic curvature κ_g of a rod (see (11)). Evolution equations, or growth laws, of this type play a central role in biomechanical models for growth, and our equation for κ_g has similarities to related works on the growth development in continuum theories (see Taber (1995, 2009) and references cited therein). The Eq. (11) enables the rod to be used to model growing plant stems and we use it in conjunction with an earlier theory that we developed in Faruk Senan et al. (2008). This theory, which is based on Euler's theory of the elastica, is sufficiently general to accommodate tip growth (also known as primary growth), lateral accretion (secondary growth), an evolving intrinsic curvature κ_g, and remodeling. However, the evolution equation for κ_g that is used in Faruk Senan et al. (2008) is inadequate when there is significant tip growth. The growth law presented in this paper addresses this deficiency and we illustrate its use with a range of examples which include comparisons to earlier published works on plant stem growth modeling.

The selection of rod theories to model plant stems is an obvious choice in many respects. However, several technical difficulties quickly present themselves. One of the main issues is the development of growth strains in so-called reaction woods of wooded plant stems. These strains lead to the development of residual stresses (which are often referred to as growth stresses), and the residual stresses can be released when the plant is felled and cut into planks and often leads to distortion of the planks. There has been a considerable volume of work aimed at quantifying and modeling these residual stresses and the biochemical stimuli that control them (see Archer (1986), Fournier et al. (1990), Ormarsson

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