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# A coupled discrete/continuous method for computing lattices. Application to a masonry-like structure

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ABSTRACT

This paper presents a coupled discrete/continuous method for computing lattices and its application to a masonry-like structure. This method was proposed and validated in the case of a one dimensional (1D) railway track example presented in Hammoud et al. (2010). We study here a 2D model which consists of a regular lattice of square rigid grains interacting by their elastic interfaces in order to prove the feasibility and the robustness of our coupled method and highlight its advantages. Two models have been developed, a discrete one and a continuous one. In the discrete model, the grains which form the lattice are considered as rigid bodies connected by elastic interfaces (elastic thin joints). In other words, the lattice is seen as a "skeleton" in which the interactions between the rigid grains are represented by forces and moments which depend on their relative displacements and rotations. The continuous model is based on the homogenization of the discrete model (Cecchi and Sab, 2009). Considering the case of singularities within the lattice (a crack for example), we develop a coupled model which uses the discrete model in singular zones (zones where the discrete model cannot be homogenized), and the continuous model elsewhere. A new criterion of coupling is developed and applied at the interface between the discrete and the continuum zones. It verifies the convergence of the coupled solution to the discrete one and limits the size of the discrete zone. A good agreement between the full discrete model and the coupled one is obtained. By using the coupled model, an important reduction in the number of degrees of freedom and in the computation time compared to that needed for the discrete approach, is observed.



## 1. Introduction

The aim of this paper is to propose an extension to 2D structures of the 1D coupled method between discrete and continuum media proposed in Hammoud et al. (2010). We focus here on the robustness and the feasibility of the coupled method in the presence of cracks and stress concentrations.

Actually, the 1D model studied in Hammoud et al. (2010) consisted of a beam resting on an elastic springs. The deflection of the beam (as well as the nodal parameters) was calculated by using two approaches; a discrete approach and a macroscopic approach deduced from the discrete one. A comparison between the response of the system obtained by using these approaches showed the cases where the macroscopic approach cannot replace the discrete one. This difference leaded us to apply a discrete/continuum coupling method. A new criterion of coupling was developed and applied at the interface of the discrete and continuum subdomains. In the coupled approach, the macroscopic scale was the intial scale computation. A local discrete computation was done on each macroscopic element. A comparison was done between the nodal parameters computed by the local discrete method and the continuum one. If a strong difference was observed, a refinement of the computation scale was done. This procedure of refinement was necessary in the zone of singularities.

In this present research, a 2D model will be considered. A masonry pannel can be described by a discrete model or a continuous model. See Alpa and Monetto (1994), Sab (1996), Cecchi and Sab (2004), Cecchi and Sab (2004) and Cluni and Gusella (2004), for example. In the discrete model, the blocks which form the masonry wall are modeled as rigid bodies connected by elastic interfaces. Then, the masonry is seen as "skeleton" in which the interactions between the rigid blocks are represented by forces and moments which depend on their relative displacements and rotations. The second model is a continuous one based on the homogenization of the discrete model. The aim of this paper is to extend the 1D coupled method of Hammoud et al. (2010) to 2D structures in the presence of cracks and stress concentrations.

Many coupled approaches between discrete and continuum media were developed. See among others the works of Broughton et al. (1999), Curtin and Miller (2003), Wagner and Liu (2003),

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