International Journal of Solids and Structures 48 (2011) 2630–2641

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

The problem of sharp notch in couple-stress elasticity

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ARTICLE INFO

Article history:
Received 4 March 2011
Received in revised form 11 May 2011
Available online 24 May 2011

Keywords:
Sharp notch
Cracks
Granular media
Microstructure
Micro-mechanics
Couple-stress elasticity
Asymptotic analysis
Boundary layer

ABSTRACT

The problem of sharp notch in couple-stress elasticity is considered in this paper. The problem involves a sharp notch in a body of infinite extent. The body has microstructural properties, which are assumed to be characterized by couple-stress effects. Both symmetric and anti-symmetric loadings at remote regions are considered under plane-strain conditions. The faces of the notch are considered traction free. To determine the field around the tip of the notch, a boundary-layer approach is followed by considering an expansion of the displacements in a form of separated variables in a polar coordinate system. Our analysis is in the spirit of the Knein–Williams and Karp–Karal asymptotic techniques but it is much more involved than its corresponding analysis of standard elasticity due to the complicated boundary value problem (higher-order system of governing PDEs and additional boundary conditions as compared to the standard theory). Eventually, an eigenvalue problem is formulated and this, along with the restriction of a bounded potential energy, provides the asymptotic fields. The cases of a crack and a half-space are analyzed as limit cases of the general notch problem. Certain deviations from the standard classical elasticity results are noted.

1. Introduction

The present work is concerned with the determination of the asymptotic displacement, rotation, strain and stress fields in the vicinity of the tip of a notch within the framework of couple-stress elasticity. This theory assumes that, within an elastic body, the surfaces of each material element are subjected not only to normal and tangential forces but also to moments per unit area. The latter are called *couple-stresses*. Such an assumption is appropriate for materials with granular structure, where the interaction between adjacent elements may introduce internal moments. In this way, characteristic material lengths appear representing the microstructure. As is well-known, the fundamental concepts of the couple-stress theory were first introduced by Voigt (1887) and the Cosserat brothers (1909), but the subject was generalized and reached maturity only in the 1960s with the studies of Toupin (1962), Mindlin and Tiersten (1962), and Koiter (1964).

The theory of couple-stress elasticity assumes that: (i) each material particle has three degrees of freedom, (ii) an augmented form of the Euler–Cauchy principle with a non-vanishing couple traction prevails, and (iii) the strain-energy density depends upon both the strain and the gradient of rotation. The theory is different from the general Cosserat (or micropolar) theory that takes material particles with six independent degrees of freedom (three displacement components and three rotation components, the latter involving rotation of a micro-medium w.r.t. its surrounding medium). Sometimes, the name 'restricted Cosserat theory' appears in the literature for the couple-stress theory.

Couple-stress elasticity had already some successful applications in the 1960s and 1970s mainly on stress-concentration problems concerning holes and inclusions (see e.g. Mindlin, 1963; Weitsman, 1965; Bogy and Sternberg, 1967; Hsu et al., 1972; Takeuti and Noda, 1973; Itou, 1976). In recent years, the couple-stress theory (and related generalized continuum theories) attracted a renewed and growing interest in dealing with problems of microstructured materials. For instance, problems of dislocations, plasticity, fracture and wave propagation have been analyzed within the framework of couple-stress theory. This is due to the inability of the classical theory to predict the experimentally observed size effect and also due to the increasing demand to study problems at very small scales. Work along these lines was done by, among others, Fleck et al. (1994), Vardoulakis and Sulem (1995), Lakes (1995), Huang et al. (1997, 1999), Lubarda and Markenskoff (2000), Bardet and Vardoulakis (2001), Georgiadis and Velgaki (2003), Lubarda (2003), Radi (2007, 2008), and Gourgiotis and Georgiadis (2007, 2008).

For materials with microstructure, the characteristic material length mentioned before may be on the same order as the length of the microstructure. For instance, Chen et al. (1998) developed a continuum model for cellular materials and found that the continuum description of these materials obey a gradient elasticity theory of the couple-stress type. In the latter study, the intrinsic material length was naturally identified with the cell size. Also,

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doi:10.1016/j.ijsolstr.2011.05.010