International Journal of Solids and Structures 48 (2011) 1154–1163

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Identification of an ellipsoidal defect in an elastic solid using boundary measurements

E.I. Shifrin *, P.S. Shushpannikov

A. Yu. Ishlinsky Institute for Problems in Mechanics, Russian Academy of Sciences, Prosp. Vernadskogo 101-1, Moscow 119526, Russia

ARTICLE INFO

Article history:
Received 19 July 2010
Received in revised form 13 November 2010
Available online 30 December 2010

Keywords:
Elastostatic inverse problem
Ellipsoidal defect
Reciprocity gap functional

ABSTRACT

Elastostatic problem of identification of an ellipsoidal cavity or inclusion (rigid or linear elastic) in an isotropic, linear elastic solid is considered. The reciprocity gap functional method is used for solving the problem. It is shown that the parameters of the ellipsoidal defect (coordinates of its center, the directions and magnitudes of the semiaxes and elastic moduli in the case of isotropic, linear elastic inclusion), located in an infinite elastic solid are expressed by means of the values of the reciprocity gap functional. The values of the reciprocity gap functional can be calculated if the loads and displacements corresponding to uniaxial tension (compression) of an infinite solid are known on the closed surface containing the defect inside. Applications of the results to the problem of ellipsoidal defect identification in a bounded body are discussed. A number of numerical examples showing the efficiency of the developed identification method are considered.

1. Introduction

The reciprocity gap functional (RGF) is defined as an integral over a closed surface, located in an elastic body, of the function, depending on the given and auxiliary elastic fields. The RGF is equal to zero for all regular auxiliary elastic fields if there are no defects inside the surface. If a defect is located inside the closed surface, then the values of RGF can differ from zero for some regular auxiliary elastic fields and the values of RGF give information about the defect. Similar properties of the scalar elliptic equations were used for solving inverse problems by Andrieux and Ben Abda (1996), Bannour et al. (1997), El Badia and Ha-Duong (2000), Alves et al. (2004), El Badia (2005). Andrieux et al. (1999) applied the RGF method for solving elastostatic inverse problem of a plane crack identification. Other publications concerning applications of the RGF method to the inverse problems can be found in the reviews of Bonnet and Constantinescu (2005) and Avril et al. (2008). The RGF method enables also to develop an analytical approach for defect parameters identification in some particular cases. The problems of identification of spherical and spheroidal defects in an elastic solid were solved analytically in Goldstein et al. (2007) and Shifrin and Shushpannikov (2010). Solutions of the problems used substantially the spherical and axial symmetry of the defects, respectively. Shifrin (2010) proposed an approach for determination of the geometrical parameters of an arbitrary ellipsoidal defect using results of one uniaxial tension (compression) test. The further development of the approach is presented in this paper. The formulas for determination of elastic constants of an isotropic, linear elastic inclusion are obtained also. A number of numerical examples, including the cases when a defect has non-ellipsoidal shape, are considered. The examples show that developed identification method enables to determine the parameters of an ellipsoidal defect with high accuracy. In the case of a non-ellipsoidal defect the method enables to construct an ellipsoid that reasonably approximates the defect.

2. Statement of the problem

Let $V \subset R^3$ be a simply connected domain with a boundary ∂V, and $G \subset V$ is an ellipsoid, $\Omega = V \backslash G$. Let us suppose that an isotropic, linear elastic body with a shear modulus μ_M and Poisson ratio ν_M occupies the domain Ω. The ellipsoidal defect G can be a cavity or an inclusion (rigid or linear elastic). Let us introduce Cartesian coordinates $OX_1X_2X_3$. We will mark with the superscript d the stress–strain state in the body $\Omega : \sigma^d_{ij}$ is the stress tensor, e^d_{ij} is the strain tensor and $\mathbf{u}^d = (u^d_1, u^d_2, u^d_3)$ is the displacement vector. According to our suppositions the following equalities are valid for $X = (X_1, X_2, X_3) \in \Omega$:

$$e^d_{ij} = \frac{1}{2}\left(u^d_{i,j} + u^d_{j,i}\right), \quad (i = 1, 2, 3;\ j = 1, 2, 3)$$
$$\sigma^d_{ij} = 2\mu_M\left[\frac{\nu_M}{1 - 2\nu_M}\theta^d\delta_{ij} + e^d_{ij}\right], \quad \theta^d = \sum_{k=1}^{3} e^d_{kk} \qquad (1)$$
$$\sigma^d_{ij,j} = 0$$

* Corresponding author. Tel.: +7 495 4343665; fax: +7 499 7399531.
E-mail addresses: shifrin@ipmnet.ru (E.I. Shifrin), shushpan@ipmnet.ru (P.S. Shushpannikov).

0020-7683/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijsolstr.2010.12.016