



Singular boundary method for solving plane strain elastostatic problems

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ABSTRACT

This study documents the first attempt to apply the singular boundary method (SBM), a novel boundary only collocation method, to two-dimensional (2D) elasticity problems. Unlike the method of fundamental solutions (MFS), the source points coincide with the collocation points on the physical boundary by using an inverse interpolation technique to regularize the singularity of the fundamental solution of the equation governing the problems of interest. Three benchmark elasticity problems are tested to demonstrate the feasibility and accuracy of the proposed method through detailed comparisons with the MFS, boundary element method (BEM), and finite element method (FEM).

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1. Introduction

The finite element method (FEM) has long been a dominant numerical technique in the simulation of elasticity problems. However, this method requires tedious domain meshing which is often computationally costly and sometimes mathematically troublesome (Brebbia, 1978), especially when the geometry of the body is not simple. As a domain discretization technique, the FEM is also less effective for inverse problems in which measurement is often only accessible on the boundary. As an alternative approach, the boundary element method (BEM) has long been touted to avoid such drawbacks (Brebbia, 1978; Cheng and Cheng, 2005; Cruse, 1988). During the past two decades, this method has rapidly improved, and is nowadays considered as a competing method to FEM. Despite the fact that the BEM requires only meshing on the boundary, it involves quite sophisticated mathematics and some difficult numerical integrations of singular functions. It is also worth noting that the discretization matrix of the BEM is fully populated due to its global approximation, and thus the total computational costs are not as low as expected compared to the costs associated with the local FEM, which results in a sparse matrix. Moreover, surface meshing in a three-dimensional (3D) domain is still a nontrivial task.

Thus, over the past decade, some considerable effort was devoted to eliminating the need for meshing. This led to the development of meshless methods which require neither domain nor boundary meshing. They still require discretizations via sets of nodes, but these nodes need not have any connectivity, and the trial functions are built entirely in terms of nodes. Among these methods, the method of fundamental solutions (MFS) has emerged as a boundary only collocation method with the merit of easy programming, high accuracy, and fast convergence (Chen et al., 1998; Fairweather and Karageorghis, 1998; Karageorghis, 1992). During the past decade, this method has been successfully applied to the solution of plane isotropic elasticity problems (Liu, 2002; Marin and Lesnic, 2004; Redekop, 1982), axisymmetric problems in elastostatics (Redekop and Thompson, 1983), 3D problems in elastostatics (Redekop and Cheung, 1987), and to problems associated with layered elastic materials (Berger and Karageorghis, 2001).

In the traditional MFS, a fictitious boundary slightly outside the problem domain is required in order to place the source points and avoid the singularity of the fundamental solution. Despite many years of focused research, the determination of the distance between the real boundary and the fictitious boundary is based on experience and therefore troublesome (Cheng et al., 2000; Liu, 2010). In recent years, various efforts have been made aiming to remove this barrier in the MFS, so that the source points can be placed on the real boundary directly. These methods include, but are not limited to, the boundary knot method (BKM) (Chen and Hon, 2003; Chen and Tanaka, 2002); the boundary particle method (BPM) (Chen and Fu, 2009; Fu et al., 2009); the boundary collocation method (BCM) (Chen et al., 2002, 2004); the regularized

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