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Large deformation and stability of an extensible elastica with an unknown length

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ABSTRACT

A beam resting on spatially fixed supports may slide relatively to these as soon as external forces are applied. Consequently, the length of the portion of the reference configuration, which is currently located between the supports, depends on the loading and therefore is not known in advance. In the present paper, the problem of a slender beam under a uniformly distributed force is investigated, which is clamped at one side but may slide through another clamping device in axial direction at the opposite side. In combination with a suitable coordinate transformation by which the numerical treatment is simplified, a finite element approach is utilized to determine the equilibrium shape for the maximum critical load that can be imposed on the beam. In the course of this, the influence of the extensibility of the beam axis is studied. A theory based on Reissner's geometrically exact relations for the plane deformation of beams is adapted such that it allows constitutive relations on stress–strain level to be integrated consistently. In addition to the classical equations of the extensible elastica, a constitutive model derived from the St. Venant–Kirchhoff material of non-linear continuum mechanics is studied. The results obtained in this survey are finally compared to those from the linear beam theory, which turns out to be incapable of describing the problem under consideration in a satisfactory manner.

1. Introduction

In the context of rod-like structures, the term stability often refers to problems of buckling, in which the equilibrium path may bifurcate as soon as the applied load exceeds a critical value. An extensive overview on various aspects of this topic is provided in Simitses and Hodges (2006). In the present work, a problem of bending is investigated in which a critical load can also be found, but—in contrast to buckling—equilibrium does not exist beyond this load. Consider a beam, which is clamped at the left-hand side and has another clamping device applied at the right-hand side at a spatially fixed point. This device allows the beam to slide through freely in axial direction but inhibits the rotation of the cross-section currently located there, see Fig. 1. Briefly speaking, the beam may slip out of an orifice at the right side as soon as forces are applied to it. The length of that part of the beam which is currently situated in between the two clamping devices depends on the applied forces and therefore is not known in advance. Neither is the cross-section which is currently located at the clamping device on the right-hand side. Examples for settings of this type include, for instance, hot and cold rolling processes in metal forming.

In literature, problems of this kind are sometimes referred to as variable-arc-length beams (Chucheepsakul et al., 1995). A boundary setting as described above, however, has not been reported so far. Most of the preceding work deals with beams for which one end is hinged while the other one may slide freely over a frictionless point support. For this type of boundaries, beams subjected to terminal couples (Chucheepsakul et al., 1995, 1994, 1999), as well as concentrated forces, both in a fixed direction (Wang et al., 1997) and as a follower force (Chucheepsakul and Phungpaigram, 2004), have been investigated by the research group of Chucheepsakul. More recently, the studies have been extended to the static and dynamic behavior of beams under self-weight, which have been analyzed both analytically and experimentally (Pulngern et al., 2005b,a). A slightly modified boundary setting with one support elevated above the other has been investigated by Athisakul and Chucheepsakul (2008). The stability of a beam under a concentrated force, which is clamped at one end while sliding over a point support at the other, has been studied by Zhang and Yang (2005). The slip-through of a beam under self-weight resting on two point supports has been examined by Chen et al. (2010). With the particular choice of boundary conditions in the present paper, the part of the beam in between the clamping devices is fully decoupled from the outside part without any idealizing assumptions being necessary, in contrast to those cases, in which the beam slides over a point support. A similar setting can be found in the contribution by Ro et al. (2010) on the vibration and stability of a beam with variable length under a compressive force, for which the buckling is constrained by walls parallel to the undeformed beam axis. For a closely related problem, in which the position of an intermediate support with respect to the

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