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# Interactive buckling regarding the axial extension mode of a thin-walled channel under uniform compression in the first nonlinear approximation

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### ABSTRACT

The present paper deals with an influence of the axial extension mode on interactive buckling of a thinwalled channel with imperfections subjected to uniform compression, when a shear lag phenomenon and distortional deformations are taken into account. A plate model is adopted for the channel. The structure is assumed to be simply supported at the ends. A method of the modal solution to the coupled buckling problem within the first order approximation of Koiter's asymptotic theory, using the transition matrix method and Godunov's orthogonalization (Kolakowski and Krolak, 2006), has been used. The calculations have been carried out for a thin-walled channel.

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### 1. Introduction

Thin-walled structures composed of plate elements have many different buckling modes that vary in quantitative (e.g., a number of halfwaves) and qualitative (e.g., global and local buckling) aspects. The analysis of buckling in conservative systems belongs to the main problems that have been studied in mechanical sciences for a number of years. In the case of finite displacements, different buckling modes are interrelated even with the loads close to their critical values (eigenvalues of the respective boundary problem). The investigation of stability of equilibrium states requires an application of a nonlinear theory that enables estimation of an influence of different factors on the structure behavior. In these cases, the postcritical behavior cannot be described any more by a single generalized displacement. When the postcritical behavior of each individual mode is stable, the mode interaction can lead to unstable behavior, and thus to an increase in the imperfection sensitivity (Ali and Sridharan, 1988; Benito and Sridharan, 1984-1985; Kolakowski and Krolak, 1995; Kolakowski, 1987, 1989a,b, 1993a,b; Kolakowski and Teter, 1995; Krolak and Kolakowski, 1995; Manevich, 1985, 1988; Manevich and Kolakowski, 1997; Moellmann and Goltermann, 1989; Pignataro and Luongo, 1987; Pignataro et al., 1985; Sridharan and Ali, 1986; Sridharan and Benito, 1984; Sridharan and Peng, 1989). A nonlinear stability theory should describe all modes of global, local, distortional and interactive buckling, taking into consideration the structure imperfections as well.

The concept of interactive buckling (the so-called coupled buckling) involves the general asymptotic theory of stability (Thompson and Hunt, 1973; Budiansky, 1974). Among all versions of the general nonlinear theory, Koiter's theory (Koiter, 1976; Koiter and Pignataro, 1976; Koiter and van der Neut, 1980) of conservative systems is the most popular one, due to its general character and development, even more so after Byskov and Hutchinson (1977) derived a complete set of formulas for the post-buckling constants associated with an interaction between modes. The theory is based on asymptotic expansions of the post-buckling path and is capable of considering nearly simultaneous buckling modes.

The expression for potential energy of the system is expanded into a series relative to the amplitudes of linear modes near the point of bifurcation, which generally corresponds to the minimum value of critical load (the so-called bifurcation load). In the potential energy expression for the first order nonlinear approximation, the coefficients of cubic terms are the key terms governing the interaction.

In the case the critical values corresponding to global buckling modes are significantly lower than local modes, their interaction can be considered within the first nonlinear approximation (Byskov, 1987–1988; Kolakowski and Kowal-Michalska, 1999; Kolakowski and Teter, 2000; Kolakowski, 1993; Kolakowski et al., 1999; Sridharan and Ali, 1986; Sridharan and Peng, 1989). It is possible as the post-buckling coefficient for uncoupled buckling is equal to zero for the second order global mode in the Euler column model, and it is very often of little significance in the case of an exact solution. The theoretical load-carrying capacity, obtained

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