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A Timoshenko beam theory with pressure corrections for layered orthotropic beams

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ABSTRACT

A Timoshenko beam theory for layered orthotropic beams is presented. The theory consists of a novel combination of three key components: average displacement and rotation variables that provide the kinematic description of the beam, stress and strain moments used to represent the average stress and strain state in the beam, and the use of exact axially-invariant plane stress solutions to calibrate the relationships between all these quantities. These axially-invariant solutions, which we call the fundamental states, are also used to determine a shear strain correction factor as well as corrections to account for effects produced by externally-applied loads. The shear strain correction factor and the external load corrections are computed for a beam composed of isotropic layers. The proposed theory yields Cowper's shear correction for a single isotropic layer, while for multiple layers new expressions for the shear correction factor are obtained. A body-force correction is shown to account for the difference between Cowper's shear correction and the factor originally proposed by Timoshenko. Numerical comparisons between the theory and finite-elements results show good agreement.

1. Introduction

The equations of motion for a deep beam that include the effects of shear deformation and rotary inertia were first derived by Timoshenko (1921, 1922). Two essential aspects of Timoshenko's beam theory are the treatment of shear deformation by the introduction of a mid-plane rotation variable, and the use of a shear correction factor. The definition and value of the shear correction factor have been the subject of numerous research papers, some of which are discussed below. Shames and Dym (1985, Ch. 4, p. 197) provide an excellent overview of the classical approach to Timoshenko beam theory. This paper however, draws primarily from research and theories which refine Timoshenko's original approximations.

Prescott (1942) derived the equations of vibration for thin rods using average through-thickness displacement and average rotation variables. He introduced a shear correction factor to account for the difference between the average shear on a cross section and the expected quadratic distribution of shear.

Cowper (1966) presented a revised derivation of Timoshenko's beam theory starting from the equations of linear elasticity for a prismatic, isotropic beam in static equilibrium. Cowper introduced residual displacement terms that he defined as the difference between the actual displacement in the beam and the average displacement representation. These residual displacements account for the difference between the average shear strain and the shear strain distribution. Cowper introduced a correction factor to account for this difference and computed its value based on the three-dimensional solution of a cantilever beam subjected to a tip load.

Stephen and Levinson (1979) developed a beam theory along the lines of Cowper's, but recognized that the variation in shear along the length of the beam would lead to a modification of the relationship between bending moment and rotation. This variation had been neglected by Cowper.

Following the work of Cowper (1966) and Stephen and Levinson (1979), in this paper we seek a solution to a beam problem based on average through-thickness displacement and rotation variables. In a departure from previous work, we introduce *strain moments*, which are analogous to the stress moments used in the equilibrium equations. These strain moments remove the restriction of working with an isotropic, homogeneous beam. This is an essential component of the present approach, as sandwich and layered orthotropic beams are often used for high-performance, aerospace applications (Flower and Soutis, 2003).

Another important feature of our theory is the use of certain statically determinate beam problems that we use to construct the relationship between stress and strain moments, and to reconstruct the stress and strain solution in a post-processing step. We call these solutions the *fundamental states* of the beam. The present theory was first pursued by Hansen and Almeida (2001) and Hansen et al. (2005), and an extension of this theory to the analysis

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