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On the thermomechanical coupling in finite strain plasticity theory with non-linear kinematic hardening by means of incremental energy minimization

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ABSTRACT

The thermomechanical coupling in finite strain plasticity theory with non-linear kinematic hardening is analyzed within the present paper. This coupling is of utmost importance in many applications, e.g., in those showing low cycle fatigue (LCF) under large strain amplitudes. Since the by now classical thermomechanical coupling originally proposed by Taylor and Quinney cannot be used directly in case of kinematic hardening, the change in heat as a result of plastic deformation is computed by applying the first law of thermodynamics. Based on this balance law, together with a finite strain plasticity model, a novel variationally consistent method is elaborated. Within this method and following Stainier and Ortiz (2010), all unknown variables are jointly and conveniently computed by minimizing an incrementally defined potential. In sharp contrast to previously published works, the evolution equations are a priori enforced by employing a suitable parameterization of the flow rule and the evolution equations. The advantages of this parameterization are, at least, twofold. First, it leads eventually to an unconstrained stationarity problem which can be directly applied to any yield function being positively homogeneous of degree one, i.e., the approach shows a broad range of application. Secondly, the parameterization provides enough flexibility even for a broad range of non-associative models such as kinematic hardening of Armstrong–Frederick-type. Different to Stainier and Ortiz (2010), the continuous variational problem is approximated by a standard, fully-implicit time integration. The applicability of the resulting numerical implementation is finally demonstrated by analyzing the thermodynamically coupled response for a loading cycle.

1. Introduction

Focusing on the framework of classical continuum mechanics, isothermal finite strain plasticity theory is nowadays relatively well developed and reasonably well understood. In case of isotropic models, the reader is referred to the comprehensive overview (Simo, 1998) and references cited therein. Currently, active research is shifting towards more realistic hardening models (see, e.g., (Wang et al., 2008)) as well as to the mathematical structure of finite strain plasticity theory, cf. (Mielke, 2004). Surprisingly, the opposite is true in case of the fully thermomechanically coupled problem. Here, only relatively simple models have been considered so far. For instance, for the temperature induced by plastic deformation, the over 70 years old purely empirical rule advocated in the pioneering work of Taylor and Quinney (1934) is most frequently applied, cf. (Simo, 1998; Wriggers et al., 1992; Simo and Miehe, 1992). This is particularly astonishing, since the understanding of the thermomechanical coupling is of utmost importance in many applications. One such typical application is metal forming. Often, the metal workpieces are heated for shaping them more easily. Another example is fatigue induced by temperature cycles (see Sauerland and Mahnken (2009)).

A physically more sound thermomechanical coupling is provided by the first law of thermodynamics itself. Starting with this law and considering the definition of the entropy, the heat change as a result of elastic and inelastic deformation can be derived, cf. (Ibrahimbegovic and Chorfi, 2002; Canadija and Brnic, 2004; Hakansson et al., 2005; Canadija and Brnic, 2010). Although this procedure is well known, it is still not the common choice in constitutive models. This is particularly strange, since this approach is thermodynamically consistent and thus, it can be applied to every hardening rule. By way of contrast, the classical Tayloy–Quinney coupling can violate the second law of thermodynamics, e.g., in case of kinematic hardening, cf. (Hakansson et al., 2005; Chaboche, 1993; Chaboche, 1993). Furthermore, experimental observations clearly show that the portion of the plastic stress power which transforms to heat is not constant in general as assumed in Taylor and Quinney (1934) (see Rosakis et al., 2000; Hodowany et al., 2000; Oliferuk et al., 2004) for further experiments; extended

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