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Stress concentration around a hole in a radially inhomogeneous plate

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ABSTRACT

The stress concentration factor around a circular hole in an infinite plate subjected to uniform biaxial tension and pure shear is considered. The plate is made of a functionally graded material where both Young's modulus and Poisson's ratio vary in the radial direction. For plane stress conditions, the governing differential equation for the stress function is derived and solved. A general form for the stress concentration factor in case of biaxial tension is presented. Using a Frobenius series solution, the stress concentration factor is calculated for pure shear case. The stress concentration factor for uniaxial tension is then obtained by superposition of these two modes. The effect of nonhomogeneous stiffness and varying Poisson's ratio upon the stress concentration factors are analyzed. A reasonable approximation in the practical range of Young's modulus is obtained for the stress concentration factor in pure shear loading.

1. Introduction

Functionally graded materials (FGMs) were initially brought to scientific attention in 1984 (Koizumi, 1997). These nonhomogeneous composites are used in various applications (see, e.g. Watari et al., 1997; Simonet et al., 2007). The composition and the morphology of FGMs gradually change over the volume; consequently, the elastic properties of the material change with position (Miyamoto et al., 1999). Of particular interest in this contribution is the special case where the elastic properties within an elastic body vary in the radial direction but are independent of tangential direction. This type of inhomogeneity can be due to several causes: directional cooling leading to a microstructural gradient (Markworth et al., 1995); phase segregation arising as a result of centrifugal casting (Fukui et al., 1999); and surface modification using laser technology (Islam, 1996).

The elastic stress field in FGMs, where the elastic properties vary in the radial direction, has recently received considerable attention (Horgan and Chan, 1999; Noda, 1999; Zimmerman and Lutz, 1999; Dryden and Jayaraman, 2006). For these types of FGMs most of the analytic investigation has been axisymmetric and directed toward pressurized FG tubes and disks (Tutuncu, 2007; Mohammadi and Dryden, 2009), curved beams (Kardomateas, 1990; Dryden, 2007; Mohammadi and Dryden, 2008), and thermal stress analysis (Jabbari et al., 2002; Lutz and Zimmerman, 1996a).

There has been less analytical investigation on the effect of radial inhomogeneity upon the elastic field in non-axisymmetric problems (see, e.g. Batra and Nie, 2009; Nie and Batra, 2009). Curved beams have been investigated by Lekhnitskii (1981). Other geometries such as hollow cylinders by Shao et al. (2008), spheres by Poultangari et al. (2008), pressurized vessels by Jabbari et al. (2003), and spherical inclusions by Lutz and Zimmerman (1996b). In most of these contributions power law functions have been used to define the variation of elastic properties. Here we want to calculate the stress field around a circular hole subjected to uniform far-field stress and power law functions are not suitable to define the spatial variation of Young's modulus in an infinite plate.

The stress concentration factor around a hole in a homogeneous plate has been received much attention over the last two centuries (see, e.g. Savin, 1961; Pilkey and Pilkey, 2008). In the case of functionally graded materials, some numerical work has been done recently. Using an isoparametric finite element formulation, Kubair and Bhanu-Chandar (2008) investigated stress concentration around a circular hole in functionally graded panels under uniaxial tension. They found that the stress concentration factor is reduced when Young's modulus progressively decreases towards the hole. Subsequently, Yang et al. (2009) investigated the stress field around a circular hole in a FGM plate. They used piece-wise homogeneous layers and complex variable methods. The plate was decomposed into N rings with equal thickness and constant material properties. The elastic fields for different spatial variations of the elastic properties were calculated.

The aim of this contribution is to present an analytical calculation for the stress concentration factor around a circular hole in an infinite plate made of an inhomogeneous material subjected to uniform biaxial tension and pure shear loading. Here, both Young's modulus and Poisson's ratio are smooth monotonic functions of the radius and attain limiting values far from the hole. Exponential

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