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On the asymptotic boundary conditions of an anisotropic beam via virtual work principle

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ABSTRACT

In this research, an efficient and effective method is proposed to derive the boundary conditions of an anisotropic beam in the asymptotic sense. We first set up the constrained virtual work by introducing the Lagrange multiplier on the displacement prescribed boundary. The macroscopic beam and microscopic cross-section equations with the boundary conditions are simultaneously obtained by taking the asymptotic expansion on the displacement vector. In this way, the three-dimensional characteristics of the beam are asymptotically smeared into the macroscopic beam equations and the beam boundary conditions. The boundary conditions obtained are then compared to those from the decay analysis method. The beam bending slope boundary condition obtained in the frame work of variational principle is different from the well-known average condition. This new boundary condition is more accurate than the average one for a sandwich beam. This is further demonstrated and discussed via the examples of a cantilever beam loaded at the end.

1. Introduction

An asymptotic method is mathematically rigorous and can potentially provide accurate predictions for anisotropic heterogenous beam structures without making prior assumptions. Some of the known asymptotic methods include the formal asymptotic method (Fan and Widera, 1992; Trabucho and Viaño, 1996), the variational asymptotic method (Trabucho and Viaño, 1996; Berdichevsky, 1981), and the partial homogenization method (Panasenko, 2002). These asymptotic methods work very well for the classical approximation to the problem (e.g. Euler–Bernoulli beam theory). Although the classical beam theory is adequate for many engineering applications, one may need to obtain higher precision for other applications, such as for beam vibration with composite couplings or for sandwich beam analysis. It is however difficult to use the asymptotic methods beyond the classical approximation without relying on the boundary layer solutions (Gregory and Wan, 1984).

Various approaches have been proposed to circumvent such a difficulty, such as the formal asymptotic approach with the decay analysis method (Fan and Widera, 1992; Duva and Simmonds, 1991; Buannic and Cartraud, 2001b) and the variational asymptotic approach with the Timoshenko-like energy transformation (Yu et al., 2002; Yu and Hodges, 2004). The decay analysis method has been successfully applied to a sandwich beam and to periodic heterogenous beams as well as to isotropic and orthotropic beams. It is however challenging to obtain the asymptotically correct boundary conditions via the decay analysis method (Gregory and Wan, 1984) for general engineering applications, since it requires the beam fundamental solutions of a semi-infinite beam under unit tension, bending, flexure and torsion. The variational asymptotic approach discussed in Yu et al. (2002), and Yu and Hodges (2004) is applicable to many engineering problems and can provide reliable solutions. However, there are still limitations when the method is applied to some problems, such as for a composite box beam with bending-shear coupling (Kim et al., 2008, 2011; Kim and Wang, 2010). In this case, its macroscopic form is still building upon the traditional Rankine–Timoshenko theory, and therefore, its boundary conditions are the same as those of the Rankine–Timoshenko theory.

Given the challenges mentioned above, it would be most desirable to obtain the boundary conditions that satisfy both asymptotically correctness and engineering applicability. This can be achieved by deriving a set of simplified boundary conditions instead of using the decay analysis method. The necessary conditions to decay for four sets of boundary conditions were derived for orthotropic beams (Horgan and Simmonds, 1991), which are equivalent to the so-called averaged boundary conditions. Recently these conditions were generalized by employing the orthogonality conditions of the asymptotic displacements to the asymptotic

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