International Journal of Solids and Structures 48 (2011) 753–763

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

New integral formulation and self-consistent modeling of elastic–viscoplastic heterogeneous materials

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ARTICLE INFO

Article history:
Received 17 June 2010
Received in revised form 26 October 2010
Available online 12 November 2010

Keywords:
Heterogeneous materials
Elastic–viscoplasticity
Integral equation
Self-consistent approximation

ABSTRACT

Predicting the overall behavior of heterogeneous materials, from their local properties at the scale of heterogeneities, represents a critical step in the design and modeling of new materials. Within this framework, an internal variables approach for scale transition problem in elastic–viscoplastic case is introduced. The proposed micromechanical model is based on establishing a new system of field equations from which two Navier's equations are obtained. Combining these equations leads to a single integral equation which contains, on the one hand, modified Green operators associated with elastic and viscoplastic reference homogeneous media, and secondly, elastic and viscoplastic fluctuations. This new integral equation is thus adapted to self-consistent scale transition methods. By using the self-consistent approximation we obtain the concentration law and the overall elastic–viscoplastic behavior of the material. The model is first applied to the case of two-phase materials with isotropic, linear and compressible viscoelastic properties. Results for elastic–viscoplastic two-phase materials are also presented and compared with exact results and variational methods.

1. Introduction

Predicting the effective behavior of heterogeneous materials from the properties of the components and the microstructure represents a critical step in the design of new materials and the modeling of their mechanical behavior. In this context, micromechanical approaches are widely used to determine the macroscopic behavior of materials, from the description of its local behavior and microstructure. The micromechanical problems are generally solved by the self-consistent approximation based mainly on the integral equation introduced by Kröner (1958) and Dederichs and Zeller (1973) or on the Eshelby (1957) inclusion problem. The self-consistent method, originally proposed by Hershey (1954) and Kröner (1958) for heterogeneous elastic materials, was extended later to behavior of incremental elastic–plastic (Hill, 1965; Weng, 1981a,b; Berveiller and Zaoui, 1995) and viscous types (Hutchinson, 1976; Molinari et al., 1987).

The viscoelastic and elastic–viscoplastic cases raise a more complex problem, mainly due to the differential nature of the constitutive law which involves different orders of time derivation concerning stress and strain fields. Viscoelastic or elastic–viscoplastic self-consistent modeling is faced thus with a space/time relation, resulting from both the heterogeneous and the hereditary nature of the material. The main difficulty is to find a suitable scale transition, which would take into account the complex effect of elastic and viscous interactions, described by Suquet (1987) as the "long memory effect". Two different ways can be chosen to deal with this problem.

On the one hand, the hereditary approaches are based on the time-integral formulation of the behavior and use the Laplace–Carson transform to perform the self-consistent scheme (Laws and McLaughlin, 1978; Li and Weng, 1994a,b; Rougier et al., 1994; Masson et al., 2000; Brenner et al., 2002). In general, the methods using Laplace–Carson transform require large CPU time and memory space and are not well adapted for non-linear situations; moreover their inversions are not easy to find.

On the other hand, internal variables approaches are based on the differential formulation of the behavior (Weng, 1981a,b; Nemat-Nasser and Obata, 1986; Molinari et al., 1997; Paquin et al., 1999; Sabar et al., 2002). Theses approaches can be preferred for the simplicity of their numerical resolution. The global behavior of Representative Volume Element (RVE) is directly determined by averaging the local fields, the difficulty being mainly in the consideration of elastic–viscoplastic nature interactions. The first models (Weng, 1981a,b; Nemat-Nasser and Obata, 1986), directly deduced from the interaction law of Kröner (1961), overestimate the internal stresses. Another approach based in a complete mechanical formulation using Kunin's projection operators and translated fields, allowing self-consistent proceeding, was proposed by Paquin et al. (1999) and Sabar et al. (2002). In linear viscoelasticity, these last two approaches are in good agreement with hereditary ones,

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0020-7683/$ - see front matter
doi:10.1016/j.ijsolstr.2010.11.012