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Shear flow past slender elastic rods attached to a plane

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ABSTRACT

Shear flow past an elastic rod or a doubly periodic array of elastic rods attached to a plane is investigated with reference to flow over a ciliated surface or a carbon nanomat. In the absence of flow, the rods are straight cylinders with circular cross-sectional shape. The mathematical framework combines slender-body theory for computing the hydrodynamic load with classical beam theory for computing the rod deflection. Small deflections are computed using a finite-element method and large deflections are computed using an iterative procedure where a rod shape is assumed, the hydrodynamic load is found, and a new shape is obtained by solving a boundary-value problem involving ordinary differential equations at equilibrium. Deflected rod profiles are presented and the macroscopic slip velocity is computed in the case of flow past a doubly periodic square or triangular array of rods over a broad range of surface coverage.

1. Introduction

A flexible rod attached to a surface bends under the influence of a hydrodynamic load imparted to it by an overpassing shear flow. The rod may be isolated or belong to an ordered or random, dilute or dense collection of rods forming a ciliated surface. For example, dense populations of carbon nanotubes with typical radius 20–30 nm forming nanocarpets, nanomats, and nanoarrays can be easily produced in the laboratory using chemical vapor deposition and other techniques. Ni et al. (2008) measured the flow-induced deflection of nanotubes with approximate length $d = 40\ \mu\text{m}$ and separation $L \simeq 200\ \text{nm}$ in a nanoforest and used classical beam theory to deduce the flexural rigidity of the rolled atomic lattice. Additional interest in flow over arrays of rods and patterned surfaces has been motivated by the desire to reduce the hydrodynamic surface drag coefficient and the effective contact angle at a three-phase contact angle, and thus produce superhydrophobic substrates (e.g., Joseph et al., 2006; Rothstein, 2010).

Recently, Pozrikidis (2010) used a highly accurate boundary-integral method to study shear flow over a cylindrical rod with circular cross section of arbitrary radius attached to an infinite plane wall. The axially symmetric shape of the undeformed rod allows us to apply Fourier expansions with respect to the azimuthal angle measured around the rod axis, and thereby accurately evaluate the hydrodynamic load applied at the tip of the rod and along the cylindrical surface. By integrating the hydrodynamic traction over the circular rod contour in a horizontal plane, we obtain an effective load density along the centerline. By integrating the hydrodynamic traction over the exposed top surface of the rod, we obtain a concentrated tip load accompanied by a bending moment. Once these loads are available, the small deflection of a rod with arbitrary length to radius ratio can be computed by solving the Euler–Bernoulli equation for an elastic beam subject to a tip and distributed load.

To compute large deflections, Pozrikidis (2010) implemented an iterative procedure where a rod shape is assumed, the hydrodynamic traction is computed using a boundary-element method for Stokes flow and then projected to the centerline, a boundary-value problem for the centerline shape is solved, and a new rod shape is reconstructed by interpolation. The procedure is repeated until convergence. Reconstructing the cylindrical rod shape from the centerline profile, and *vice versa*, is an important aspect of the algorithm. Because of high computational cost, the boundary-element computations are practically limited to rods with small or moderate aspect ratio. A large number of boundary-elements is required in the case of slender rods with aspect ratio on the order of one thousand encountered in applications involving cilia and nanotubes.

To overcome these difficulties, a slender-body theory is implemented in this paper for computing Stokes flow past an attached slender rod with high aspect ratio. The hydrodynamic traction arising from the solution of an integral equation along the rod centerline is transferred to the rod as a load, and a boundary-value problem is formulated and solved for small and large deflections. Analogous numerical methods for freely suspended fibers were implemented by several previous authors. Tornberg and Shelley (2004) used slender-body theory for Stokes flow to simulate the interaction of freely suspended inextensible fibers modeled as Euler–Bernoulli beams generalized to three dimensions, satisfying

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