International Journal of Solids and Structures 48 (2011) 3336–3345

Contents lists available at SciVerse ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

A micromechanics-based strain gradient damage model for fracture prediction of brittle materials – Part I: Homogenization methodology and constitutive relations

Jia Li *

LPMTM, CNRS UPR 9001, Institut Galilée, Université Paris XIII, 99 Avenue Jean-Baptiste Clément, 93430 Villetaneuse, France

ARTICLE INFO

Article history:
Available online 22 August 2011

Keywords:
Strain gradient theory
Homogenization
Constitutive equations
Microcracks
Brittle materials
Size effect

ABSTRACT

In this paper, we first describe a homogenization methodology with the aim of establishing strain gradient constitutive relations for heterogeneous materials. The methodology presented in this work includes two main steps. The first one is the construction of the average strain-energy density for a well-chosen RVE by using a homogenization technique. The second one is the transformation of the obtained average strain-energy density to that for the continuum. An important characteristic of this method is its self-consistency with respect to the choice of the RVE: the strain gradient constitutive law built by using the present method is independent of the size and the form of the RVE. In the frame of this homogenization procedure, we have constructed a strain gradient constitutive relation for a two-dimensional elastic material with many microcracks by adopting the self-consistent scheme. It was shown that the effective behavior of cracked solids depends not only on the crack density but also on the average crack size with which the strain gradient is associated. The proposed constitutive relation provides a starting point for the development of an evolution law of damage including strain gradient effect, which will be presented in the second part of this work.

1. Introduction

In this paper, we attempt to demonstrate that strain gradient plays an important role in fracture behavior of brittle materials. We believe that through the present work, we can show an evident influence of strain gradient on the fracture of brittle materials under non-singular stress concentration. The present work is divided into two parts. In the first part, we developed a homogenization methodology with which a strain gradient constitutive relation was established for brittle materials with many microcracks. In the second part, this constitutive relation was extended into an evolution law of damage. Moreover, numerical simulations and comparative studies with experimental results were also performed.

The strain gradient constitutive laws in solid mechanics were introduced by the necessity to describe the size effect observed in micro or macro scales. This size effect involves change of response when the spatial dimensions are scaled up or down while the geometry and all other characteristics are preserved (Bazant, 1976). The size effect study in solid mechanics is a wide domain with considerable importance in engineering applications. It is a common belief that the size effect is essentially due to heterogeneities and flaws in materials. The size effect becomes noticeable when the size of these heterogeneities and flaws is comparable with respect to that of structural elements.

Generally speaking, the size effect in brittle materials has a statistical and a deterministic component. Numerous studies were reported for assessing the influence of the statistic aspect of the heterogeneities on the size effect (Carpinteri, 1994). For the deterministic component, the main difficulty resides in the fact that its study cannot readily be included in classical continuum mechanics framework. Consequently, researchers often see enriched continuum theories like non-local elasticity (e.g., Pijaudier-Cabot and Bazant, 1987) as a replacement for more complicated microscopic and discrete simulations. Under certain conditions, these non-local models can be approximated by the so-called strain gradient elasticity where higher-order gradients of strain are added to the classical elastic constitutive equations. These gradient approaches are often based on the introduction of length scale effects in elasticity, plasticity or dislocation dynamics by incorporating higher order gradients of strain into the constitutive or evolution equations governing the material description. Casal (1961, 1963, 1972) first introduced strain gradient in modeling a 1D bar behavior, and interpreted the forces corresponding to higher order strains as "capillarity forces" since they appear as important as the size of the bar becomes small. Toupin (1962) and Mindlin and Tiersten (1962) have developed the general three-dimensional theory of higher-order gradient theories for linear elastic materials. Since, the higher-order gradient theories became a popular topic of research (Mindlin, 1964, 1965; Green and Rivlin, 1964; Koiter, 1964; Kleinert, 1989; Ru and Aifantis, 1993; Aifantis, 2003; Gutkin, 2000 among others). Apart from these pure elasticity theories, the

* Tel.: +33 1 49 40 28 89; fax: +33 1 49 40 39 38.
E-mail address: jia.li@univ-paris13.fr

0020-7683/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijsolstr.2011.08.007