



On thermoelastostatics of composites with nonlocal properties of constituents

I. General representations for effective material and field parameters

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ABSTRACT

A theory of thermoelastic composites with nonlocal properties of constituents is analyzed for multiphase elastic solids of arbitrary geometry and material symmetry. Due to their generality, one uses the nonlocal integral models because the gradient models are usually derived as approximations of corresponding integral models in the immediate (infinitely closed) vicinity of the point being considered. One explores a simplified theory for linear (macroscopically) elasticity, which differs from the classical local theory in the stress–strain constitutive relation only, whereas the equilibrium and compatibility equations remain unaltered. One obtains the new representation of the effective modulus and compliance through the mechanical influence function which does not explicitly depend (as opposed to its local counterpart) on the elastic operators of constituents. The representations for the effective eigenstrains and eigenstresses through either the mechanical influence functions or transformation influence functions are presented. The effective strain energy and potential energy are expressed in terms of only average values of the state variables and the effective properties. Representations of both the first and second statistical moments of stress and strain fields in the constituents are also performed. Many of the results were obtained as the straightforward generalizations of their local counterparts because the methods used for obtaining the mentioned results widely exploit the Hill's (1963) condition which holds for any compatible strain field and equilibrium stress field not necessarily related to each other by a specific stress–strain relation.

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1. Introduction

The prediction of the behavior of composite materials in terms of the mechanical properties of constituents and their microstructure is the central focus of micromechanics, which can be reduced to the estimation of stress fields in the constituents. In so doing, in parallel with the local stress and strain fields produced by mechanical remote loading, composite materials are shown to accommodate eigenstrains or eigenstresses, and the residual fields that they generate in the constituents. Such residual stresses are of considerable interest to researches in view of their impact on the overall behavior and structural integrity of composites. Inspired by Eshelby (1957), a number of micromechanical models were proposed in the literature for describing the thermoelastic behavior of composites with ellipsoidal inclusions (see for references, e.g. Torquato, 2002; Buryachenko, 2007). Apart from these models exploiting microtopological information of the composite structure (such as, e.g., the shape of heterogeneities, their orientation and

binary correlation functions), the general results establishing the links between the effective properties (effective elastic moduli, effective thermal expansion, and effective specific heat) and the corresponding mechanical and transformation influence functions are worthy of notice. The works of this general direction were inspired by Hill (1963) and Levin (1967), who obtained the classical representations of effective elastic moduli and thermal expansion through the mechanical influence functions. Development of the mentioned methods (see Schapery, 1968; Rosen and Hashin, 1970; Laws, 1973; Kreher and Pompe, 1989; Parton and Buryachenko, 1990) achieved apparently the most generality in the works by Dvorak and Benveniste (see Dvorak, 1992; Dvorak and Benveniste, 1992; Benveniste and Dvorak, 1997), where they obtained some exact results and developed the uniform field theory to their current level.

Usually the general works in thermostatics of composites abound with studies of constitutive models for the mechanical behavior of solids. These works fall within the category of simple nonpolar materials (Noll, 1972), for which the stress at a given point uniquely depends on the deformation and temperature at that point only. However, such a consideration implicitly implies that the material can be treated as a continuum at an arbitrarily

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