International Journal of Solids and Structures 48 (2011) 2167–2176

Contents lists available at ScienceDirect

International Journal of Solids and Structures

journal homepage: www.elsevier.com/locate/ijsolstr

Tensorial characterisation of directional data in micromechanics

X. Li *, H.S. Yu

Materials, Mechanics and Structures Research Division, Process and Environmental Research Division, Faculty of Engineering, The University of Nottingham, University Park, Nottingham NG7 2RD, UK

ARTICLE INFO

Article history:
Received 23 July 2010
Received in revised form 14 February 2011
Available online 12 April 2011

Keywords:
Directional data
Statistics
Tensorial characterisation
Frame-indifferent
Multi-scale mechanics

ABSTRACT

Information being dealt with in micro-mechanics is massive. Most of them are directional data. Macro-scale physical laws embedded with micro-scale fundamentals need to be developed in terms of the statistics of the micro-scale variable in a frame-indifferent form. Mathematical techniques and theories for characterizing the statistics of directional data with tensors are hence demanded. This is the main concern of the current paper. Starting with the general theory established in Kanatani (1984) of describing the directional distributions of orientations, mathematical formulations have been extended to address the directional distributions of vector-valued directional data, which is the most common data type being dealt with in micromechanical investigations. For vector-valued directional data, statistical analyses are required in regarding to both their directional probability density distribution and their representative values along each direction. The technique used here is to approximate these directional distributions by polynomials in unit directional vector **n**. The coefficients are in tensorial form and determined from observed directional data by applying the least square error criterion. These coefficient tensors serve as macro-scale variables representing the statistics of the micro-scale directional data, and are referred to as direction tensor. Orthogonal decompositions are addressed so that the coefficient tensor of different orders can be determined independently from each other. The coefficient tensors in the orthogonal decompositions are referred to as deviatoric direction tensor. The choice of sufficient approximation order is suggested. As an example, a general form of the stress–force–fabric relationship is derived for demonstrating the application of the proposed mathematical theory in the micro-mechanical investigation of the behaviour of granular materials.

1. Introduction

Multi-scale study is a hot topic in a number of disciplines. Along with the fact that anisotropy has been recognized as an intrinsic nature of materials, massive amount of micro-scale physical quantities, being dealt with in microscopic investigations, are often direction dependent and random in nature. These micro-scale quantities generally involve measurements on both orientations and magnitudes, and present themselves in the form of vectors. Hence, statistical characterisation of directional data (Mardia, 1972) is demanded in the study of micromechanics.

The importance of mathematical techniques dealing with the statistics of directional data has been stressed by Kanatani (1984). In the study of the physical problem and for the possible application in engineering, the description of the physical laws must be presented in a frame-indifferent form, such as scalars and tensors, which are invariants to coordinate transformations. For the purpose of developing macro-scale physical laws with micro-scale mechanism properly reflected, characterizing the statistics of the directional data in terms of scalars and tensors is necessary (Cowin, 1986; Li and Dafalias, 2004; Nicot and Darve, 2005). This forms the main purpose of the current paper.

Generally speaking, there are two types of directional data. The first type is orientations, such as particle orientations, contact normal directions, and so on. Orientations are represented by unit vectors. Their magnitude is not relevant. The significant information is their directions. The second type of directional data is vectors, of which both magnitude and direction are of significance. Vector-like directional data are also the more general cases of existence. Typical examples in micro-mechanics of granular materials include contact vectors, contact forces, and particle displacements.

For orientations, the directional distribution of their probability density is of major concern. It is described by a scalar-valued direction dependent function. Kanatani (1984) proposed a theory to characterize the probability density distribution of such directional data, in terms of fabric tensors, and demonstrated its application by analyzing the data of inter-particle contact distribution of a two dimensional granular material observed in laboratory (Konishi et al., 1982).

* Corresponding author. Tel.: +44 (0)1159514167; fax: +44 (0)1159513898.
E-mail addresses: xia.li@nottingham.ac.uk, ce.lixia@gmail.com (X. Li).

0020-7683/$ - see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijsolstr.2011.03.019