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Deformation of porous Cosserat elastic bars

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ABSTRACT

This paper is concerned with the linear theory of porous Cosserat elastic solids. We study the equilibrium of a cylindrical bar which is subjected to resultant forces and resultant moments on the ends, to body loads and to surface tractions on the lateral surface. The Almansi problem, where the body loads and the surface loading on the lateral surface are polynomials in the axial coordinate, is considered. The bar is made of an inhomogeneous and isotropic material whose constitutive coefficients are independent of the axial coordinate. The problem is reduced to the study of two-dimensional problems. The results are used to study two practical applications concerning the deformation of a circular rod. It is shown that a uniform pressure on the lateral surface produces an extension, a uniform change of the porosity, and a plane deformation. The bending by terminal couples produces a non-uniform variation of the porosity and a microrotation of the material particles.

1. Introduction

In the theory of classical elasticity, the deformation of the bars has been a subject of intensive study. In recent years, research activity on the generalized theories of continuum has stimulated renewed interest in problems of elastic bars. Cosserat and Cosserat (1909) introduced a theory of mechanics of continuous media in which each material particle has the six degrees of freedom of a rigid body. The theory of Cosserat continua was extensively studied in the last decades. An account of the historical developments as well as references to various contributions may be found in the monographs by Truesdell and Toupin (1960), Kunin (1982), Nowacki (1986), Eringen (1999), Rubin (2000), Dyszlewicz (2004), Ieşan (2004). In this paper we consider the equilibrium theory of porous Cosserat elastic solids. Eringen (1990, 1999) introduced the theory of microstretch continua as a special case of the theory of micromorphic materials. A microstretch continuum is a micromorphic continuum for which the microdeformation tensor χ_{ij} has the form $\psi\delta_{ij} + \varepsilon_{ijk}\varphi_k$, where δ_{ij} is the Kronecker delta, ε_{ijk} is the alternating symbol, ψ is the microstretch function (or porosity function), and φ_k is the microrotation vector. Consequently, the microelements of a microstretch continuum undergo a uniform microdilatation characterized by ψ, and a rigid microrotation, characterized by φ_k. Nunziato and Cowin (1979) established a theory for behaviour of porous solids in which the skeletal or matrix material is elastic and the interstices are void of material. The intended applications of this theory are to geological materials like rock and soils and to manufactured porous materials like ceramics and prestressed powders. The linear theory of elastic materials with voids has been established by Cowin and Nunziato (1983). The theory of elastic materials with voids has been extensively studied (Bedford and Drumheller, 1983; Batra and Yang, 1995; Ieşan, 2004, and the literature cited therein).

It is important to note that if we neglect the microrotation vector field, then the linear equations which describe the behaviour of a microstretch elastic body coincide with the equations of an elastic material with voids established by Cowin and Nunziato (1983). In the theory of material with voids the function ψ is called volume fraction field. When ψ is zero, then the microstretch continuum model reduces to Cosserat continuum. Consequently, in the equilibrium theory, a microstretch continuum is a porous Cosserat continuum. The general theory of microstretch elasticity was presented by Eringen (1999).

In the present paper we study the deformation of porous Cosserat elastic bars in the context of the linear theory. The paper is motivated by the recent interest in the using of the Cosserat elastic continuum as model for bones (Lakes, 1986; Rosenberg et al., 2000; Fatemi et al., 2002, 2003) and for engineering materials like concrete and other composites (Masiani and Trovalusci, 1996; Diebels and Steeb, 2002; Adhikary and Guo, 2002; Casolo, 2006; Tekoglu and Onck, 2008). Lakes (1986) stated that: "Human bone, a natural fibrous composite, displays size effects in torsion and bending which are consistent with Cosserat elasticity, rather than classical elasticity". Since the cancellous bone is considered as a porous system (Kohles and Roberts, 2002; Bensamoun et al., 2004) it seems that the model studied in this paper is more adequate to describe the mechanical behaviour of bones. In this paper we study a generalization of Saint–Venant's problem to the case when the cylindrical bar is subjected to body loads and to surface forces and

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