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# Asymptotic behavior of a hard thin linear elastic interphase: An energy approach

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### 1. Introduction

During the mechanical assembly of structures, interphases can have crucial effects. In particular, imperfections in the assembly can lead to structural failure. Although the thickness of interphases is generally very small in comparison with the dimensions of the structure, their mechanical role cannot be neglected and they need to be taken into account in modeling procedures. From the numerical point of view, the thinness of interphases gives rise to problems which are very difficult to solve. In particular, the number of degrees of freedom adopted in studies using a finite element approach can be very large, which affects the convergence and the accuracy of the solution. Interphase modeling therefore has to be performed before solving the problem numerically. One classical technique consists in replacing the thin interphase by an interface of zero thickness, while keeping some important mechanical properties of the interphase. From the geometrical point of view, the interphase is eliminated, although it is accounted for mechanically. The resulting equivalent interface model is simpler to implement in numerical simulations than the original multi-scale problem. This idea was the starting-point of several studies published during the last years (Caillerie, 1980; Ait-Moussa, 1989; Klarbring, 1991; Licht, 1993; Licht and Michaille, 1996, 1997; Ould-Khaoua et al., 1996; Ganghoffer et al., 1997; Geymonat and Krasucki, 1997; Lebon et al., 1997; Zaittouni et al., 2002; Lebon and Rizzoni, 2008; Lebon and Zaittouni, 2010). To model the equivalent inter-

#### ABSTRACT

The mechanical problem of two elastic bodies separated by a thin elastic film is studied here. The stiffness of the three bodies is assumed to be similar. The asymptotic behavior of the film as its thickness tends to zero is studied using a method based on asymptotic expansions and energy minimization. Several cases of interphase material symmetry are studied (from isotropy to triclinic symmetry). In each case, non-local relations are obtained relating the jumps in the displacements and stress vector fields at order one to these fields at order zero.

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face, asymptotic techniques are necessary, i.e., we take the thickness of the interface to be a small parameter which tends to zero. Interface models usually relates the stress vector to the jump in the displacement (or in the velocity). In most cases, like in soft interface models (Geymonat et al., 1999; Krasucki et al., 2001; Lebon et al., 2004; Lebon and Ronel-Idrissi, 2004; Pelissou and Lebon, 2009; Rekik and Lebon, 2010), this means that not only the thickness of the interface but also its rigidity is small. In the present study on a hard interface model, only the thickness is assumed to be small, and the stiffness of the adherents and the interphase are taken to be similar.

Some studies, focused on adherents and a flat interphase with a comparable level of rigidity (Caillerie, 1980; Abdelmoula et al., 1998; Lebon and Ronel, 2007; Lebon and Rizzoni, 2010), have already established that at the first order ( $\varepsilon \rightarrow 0$ ) one obtains a perfect interface model, which prescribes the vanishing of the jumps in the stress and the displacement vectors. At a higher order (the second term in the expansion), an imperfect interface model is obtained, with a transmission condition involving the first order displacement and traction vectors and their derivatives (Abdelmoula et al., 1998; Lebon and Ronel, 2007; Lebon and Rizzoni, 2010). The higher order term, giving rise to an imperfect interface model, can be interpreted as a correction of the leading solution corresponding to the perfect interface model.

All these studies model the interphase as an isotropic, linear elastic material. Even though in many practical cases the adhesive is an isotropic material, typically an epoxy resin, it is possible that the process of producing a thin layer of adhesive causes the material to become anisotropic or layered. In this paper, we extend the results obtained in Abdelmoula et al. (1998), Lebon and Ronel

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