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Application of Importance Sampling As a Variance Reduction Technique in Structural Reliability

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Abstract

In structural reliability analysis, where the probability of failure is generally relatively small, the direct Monte Carlo (MC) simulation procedure becomes inefficient. In this case many simulations are required to estimate a probability and limit the uncertainty in the estimate. In MC simulation to produce defensible results for a case with a large number of random variables, a large number of sampling sets is required. Thus, there are limitations to obtaining high accuracy for large-scale problems, which require tremendous computational time and effort. To improve the rate of convergence, several modifications can be introduced to MC simulation. In this paper, Importance sampling (IS) is presented as a well-known approach in variance reduction method for MC simulation.

The importance sampling method is a modification of MC simulation in which the simulation is biased for greater efficiency. In IS, the sampling is done primarily in the tail of the distribution, rather than spreading it out evenly, in order to ensure that sufficient simulated failures occur. IS consist in generating random weighted samples from an auxiliary distribution that named IS density function rather than the distribution of interest. The crucial part of this method is the choice of IS density function that should be able to simulate more rare random events. In this paper shifted joint probability distribution function (PDF) of random variables is used for IS density function. In this method PDF shifted to the β-point (most probable of point) obtained from first-order reliability method (FORM) theory. Some examples are solved by this method and results show that the IS has smaller variance than MC with same number of simulations.

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Keywords: Monte Carlo, Importance Sampling, Reliability Analysis, probability of failure.

1. INTRODUCTION

The estimation of probability of failure generally leads to assessment of reliability of systems. Reliability also has a great role in assessment of risk in safety engineering [1,2]. Formulation of the reliability problems in the simple sense can be attributed into two essential variables of strength and load. Resistance of structure, *R* includes random variables such as stress, strain, and internal force, allowable amount of serviceability or structural capacity. The imposed loads on the structure, *Q* consist of random variables such as external loads, material properties and the geometry of a structure or the structural demand. The difference between the resistance and load can simply express as:

$$G(R,Q) = R - Q \tag{1}$$

Where *G(R,Q)* denotes the limit-state function. This function can be divided into three region; safety when (*R-Q>0*), failure region when (*R-Q<0*) and the boundary between the safety and failure when (*R-Q=0*). Reliability analysis involves calculation of probability of failure by the following integral:

$$P_{\mathrm{f}} = P(g(X) \le 0) = \int_{g(x)\le 0} f_X(x) \tag{2}$$