Portfolio optimization by minimizing bounds of loss probability

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Abstract:

Optimal alloction of capital to investment and minimizing the risk of investmen , most investors one of the main objective . The problem of allocating funds into a given set of investable assets is known as portfolio selection. In this paper, we derive a portfolio optimization model by minimizing upper and lower bounds of loss probability. Based on the bounds, two fractional programs are derived for constructing portfolios, where the numerator of the ratio in the objective includes the value-at-risk (VaR) or conditional value-at-risk (CVaR) while the denominator is any norm of portfolio vector. Some computational experiments are conducted on real stock market data, demonstrating that the CVaR-based fractional programming model outperforms the empirical probability minimization.

Keywords: Portfolio optimization, Loss probability, CVaR, Fractional programming.

1. Introduction

1.1. Portfolio selection models

The problem of allocating funds into a given set of investable assets is known as portfolio selection. Typical (single period) portfolio selection models determine the distribution of a random return of the form

$$\mathcal{R}(\boldsymbol{\pi}) := \sum_{j=1}^{n} \mathcal{R}_j \pi_j,$$

where $\mathcal{R}_j$ represents the random rate of return of asset j, and $\boldsymbol{\pi}$ represents a portfolio vector, each component representing the investment ratio into each asset. In practice, the criterion for determining a portfolio π is formulated as an optimization problem of the form

$$\min\{\mathbb{F}[\mathcal{R}(\boldsymbol{\pi})] : \boldsymbol{\pi} \in \Pi \subset \mathbb{R}^n\}, \qquad (1)$$

where the objective is a functional $\mathbb{F}$ of the random vector $\boldsymbol{\mathcal{R}} := (\mathcal{R}_1, \ldots, \mathcal{R}_n)^{\mathrm{T}}$ on a probability space $(\Omega, \mathbb{P}, \mathcal{F})$ which is independent of $\boldsymbol{\pi}$, and $\Pi \subset \mathbb{R}^n$ is a feasible region of the portfolio vector $\boldsymbol{\pi}$. By definition of $\boldsymbol{\pi}$, Π is supposed to include a constraint of the form $\boldsymbol{e}_n^{\mathrm{T}}\boldsymbol{\pi} = 1$ where $\boldsymbol{e}_n := (1, \ldots, 1)^{\mathrm{T}}$ is the n-dimensional vector of ones.

For example, the expected utility maximization criterion with some utility function U can be formulated by adopting $-\mathbb{E}\left[U\left(\mathcal{R}(\boldsymbol{x})\right)\right]$ as the objective of (1), where $\mathbb{E}[\cdot]$ denotes the mathematical expectation. More practically, a risk measure such as variance $\mathbb{V}[\mathcal{R}(\boldsymbol{\pi})]$ or a composite objective considering return as well as risk,e.g., $\mathbb{V}[\mathcal{R}(\boldsymbol{\pi})] - a\,\mathbb{E}[\mathcal{R}(\boldsymbol{\pi})]$ with a positive constant a, is preferred due to the ease in controlling the characteristics of the distribution of $\mathcal{R}(\boldsymbol{\pi})$, where $\mathbb{V}[\cdot]$ Denotes the variance operator. Alternative deviation type risk measures such as absolute deviation, $\mathbb{E}[|\mathcal{R}(\boldsymbol{\pi}) - \mathbb{E}[\mathcal{R}(\boldsymbol{\pi})]|]$, have also been used (Konno and Yamazaki,1991).