# An Efficient Numerical Approximation of the American Option Pricing Problem

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## Abstract:

This paper deals with developing an efficient numerical approximation of the American option pricing problem as a free boundary problem. The recently introduced artificial boundary conditions of Han and Wu [٩] are also employed. In order to solve the problem, a finite difference method is applied. The research has also taken advantage of the numerical approximation of the free boundary near expiry. Comparing the results coming from this method with those of the former methods, this research has been able to increase the accuracy of the commonly used methods.

**Keywords: American option, Artificial boundary condition, Finite difference method, Free boundary approximation, Partial differential equations**

## ١. Introduction

American-style options enable the holder to exercise the option at any point in time up to the maturity. The pricing and hedging of American options have been considered as a long-standing problem in computational finance calling for effective models for option pricing such as the Black-Scholes equation [٢]. Due to the complex nature of the problem and the prominent rule played by it in current financial markets, the last twenty years have witnessed an intense body of research activities to resolve the problem.

Although some analytical expressions for the price process have been obtained in the literature, there is no universal, explicit and easily computable formula currently available for the general case. This has resulted in the introduction of different analytical and semi-analytical tools to solve the problem approximately based on the Black-Scholes equation, with which the price of these options is described [٨, ١١, ١٤, ٢١]. However, these methods are along with some drawbacks which can be attributed to the fact that they could not easily be extended beyond the Black-Scholes framework.

With regard to this problem, researchers have devised and analyzed a variety of numerical approaches such as the binomial method, finite difference method, finite element method and spectral method, with which the pricing problem can be solved efficiently [٦, ٣, ١, ٤]. Besides, these approaches could be implemented in conjunction with different formulations like front-tracking transformations, front-fixing transformations, penalty methods, operator splitting ideas and singularity separating techniques [٧, ١٠, ١٣, ٥].