Robust Mean-Conditional Value at Risk Portfolio Optimization

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Abstract:

In the portfolio optimization, the goal is to distribute the fixed capital on a set of investment opportunities to maximize return while managing risk. Risk and return are quantiti es that are used as input parameters for the optimal allocation of the capital in the suggested models. But these quantities are not known at the time of the formulation and solving problem. Thus they shou ld be estimated to solve the problem which might lead to large error. One of the widely used approaches to deal with such a situation, is robust optimization. In this paper we study the mean-Conditional Value at Risk (M-CVaR) portfolio selection problems under the estimation risk in mean return for both interval and ellipsoidal uncertainty sets. Equivalent formulations of the robust counterparts are given. At end an example is given to demonstrate the impact of uncertainty.

KeyWords: Portfolio Optimization, Robust Optimization, Value at Risk, Conditional Value at Risk, Conic Optimization.

1. Introduction

The nature of the investment and business activities is such that to achieve return are required to bear the risk. Therefore, when deciding for the investment, an investor has to accept a balance between risk and return. Hence, portfolio optimization has been demonstrated as an important ploy in investment and has led to create many theories and models in this context [8].
One of the most famous theories is optimal selection of portfolio theory introduced by Harry Markowitz in 1952 [10]. The Markowitz mean-variance (MV) model has been used as the standard framework for optimal portfolio selection problems. In this model, portfolio return is represented by the expected return and risk of the portfolio is measured by the variance of the portfolio returns. The variance is a statistical dispersion measure that gives the average of the squared distance of the possible returns from the expected return. So, an asset with return better than expected return is assumed to be as risky as an asset with return lower than expected return, whereas most investors don't consider risk of the high return. Hence, the variance is an adequate measure for calculating risk only when the returns of the underlying assets are distributed symmetrically. Hence, financial institutions and individuals attention was attracted to risk management in terms of percentiles of loss distribution such as Value at Risk (VaR). Instead of regarding the whole spreading around the expected return, VaR considers only the spreading to the left of the expected return as risk and represents the predicted maximum loss with a specified confidence level (e.g. 95%) over a certain period of time (e.g. one day) [4,12].
VaR is widely used in the financial industry, but despite this popularity, it has undesirable properties that restrict its use [2, 9, 11]. One of these properties is that VaR lacks subadditivity (for a risk measure f, we should have $f(x_1+x_2) \leq f(x_1)+\mathrm{f}(x_2)$). Obviously diversification reduces risk, thus "The total risk of two different investment portfolios does not exceed the sum of the individual