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New Adaptive Monte Carlo Algorithm and Application to Financial Mathematics

R. Farnoosh **M. Aalaei***
School of Mathematics, Iran University of Science & Technology, Narmak, Tehran 16844, Iran.
E-mail: rfarnoosh@iust.ac.ir, aalaei@iust.ac.ir

Abstract
In this paper, a new adaptive Monte Carlo algorithm is proposed to solve linear systems. The proposed algorithm converges much faster than the conventional Monte Carlo algorithm. The corresponding properties of the algorithm are discussed. It has simple structure, low cost, desirable speed and accuracy.Theoretical results are established to justify the convergence of the algorithm. To confirm the accuracy and efficiency of the proposed algorithm, it is used to solve large linear systems. From the numerical results, the new adaptive Monte Carlo algorithm achieves exponential convergence. Both (the new and the old) adaptive Monte Carlo algorithms are implemented for parallel solution of large linear systems on parallel machine with MPI as inter node communication. Furthermore, we provide an application of the algorithm to price options, where the Black Scholes formula is converted to linear systems using discretization.

Keywords and phrases: Adaptive Monte Carlo algorithm, large linear systems, Parallel computing, option pricing, Black Scholes formula.

1. Introduction

High dimensional linear systems of algebraic equations are arisen from real world problems: e.g. Real-time speech coding, digital signal processing, communications, stochastic modelling, and many physical problems involving partial differential equations (see for example [1], [2], [10]) . Therefore the choice of appropriate approach for solving large sparse linear systems of algebraic equations is a problem of unquestionable importance in many scientific and engineering applications.
One of the well known stochastic algorithms which is preferable for solving high dimensional linear system of algebraic equations is Monte Carlo method. The idea is that the solution of the linear system is formulated in terms of the mathematical expectation of some random variable. Then the average of independent samples of this random variable is used to estimate the solution of the linear system.
Monte Carlo methods have three significant advantages:

1. They can approximate individual components of the solution without calculating the whole solution vector, [8].
2. For a large sparse linear system of algebraic equations, they are more efficient than direct or iterative numerical methods, [11].