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Pricing American Options by the Finite Element Method

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Abstract

In this paper we investigate the performances of high-order of finite element methods for American option pricing. First of all, the partial differential problem that yields the price of American options, which is a free boundary problem, is transformed to a problem with a fixed boundary by adding a suitable penalty term. Then, by employing a quadratic finite element method, a nonlinear system of differential equations is obtained which is solved using an ad-hoc implicit-explicit Euler time-stepping. Numerical results will be presented to demonstrate the validity and the effectiveness of the method proposed.

Keywords and phrases: American Option, Penalty Method, Finite Element Method, Explicit and Implicit Algorithm, Newton Mtheod

1. Introduction

A very popular approach to derivative pricing is the use of mathematical models based on partial differential equations. In particular, among the most commonly employed models, there is the famous Black-Scholes (BS) model. The valuation and hedging of BS equation resulting in American-style option is no doubt a challenging topic in both academic and the financial industry. This stems from the facts that most liquidly traded options are American style contracts, which allow option holders to exercise their rights before maturity, and that there is no analytical solution to these financial products under a realistic situation so far.

In this paper, we are attempting to obtain high order accuracy for the American option pricing. Following [1,2], the FEM is carried out to the nonlinear obtained B.S equation, and the variational integrals are evaluated by Gauss-Lobatto quadrature, and the initial solution is collocated at Gauss-Lobatto nodes. In addition, the computational stock and time mesh is chosen such that one of the finite element boundaries is positioned on the strike price and guarantee stability of the implicit algorithm[2]. This strategy has already proven crucial to improve the convergence rates of some lattice-based numerical methods used to solve the BS model.