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Introduction to Numerical Simulation of Stochastic Differential Equations by Using R Software and its Finantial Application

R. Lalehzari* **N. Lalehzari**† **B. Kafash**‡
Emam Javad Higher Education Institute, yazd, Iran.
Rlalehzari@gmail.com, Nlalehzari@gmail.com, Bkafash@iju.ir

Abstract
Stochastic differential equation (SDE) models play a prominent role in a range of application areas, including biology, chemistry, economics, and finance. In this paper, we will introduce numerical methods for stochastic differential equations and simulate them with `R` saftware.

Keywords and phrases: Stochastic process, Stochastic differential equation (SDE), `R` software, Stochastic simulation.

1. Introduction

In mathematical modeling, if we use stochastic systems then we will assume that the system follows a probabilistic rule and the future behavior of the system will not be known for sure. In recent years, the application of SDEs in different sciences has increased rapidly. The important difference between SDE and ordinary differential equation(ODE) is the existence of Wiener Proces. Often the analytic solution of SDEs is not available.
In this paper, we will introduce some fundamental concepts of stochastic processes and simulate some well known processes such as Brownian motion, geometric Brownian motion, stochastic integration, Ornstein-Uhlenbeck process. Also, we study the Euler-Maruyama and Milstein numerical solution of the SDE. We conclude the paper with some applications. Note that there is an article which is built around 10 MATLAB programs [1].

2. Brownian Motion and Its Simulation

A stochastic process $W(t)$ is said a standard Brownian motion on $[0,T]$ if it satisfies in some propertes [2-3]. For computational purposes it is useful to consider discretized Brownian motion, where $W(t)$ is specified at discrete t values. We thus set $\delta t = T/N$ for some positive integer N and let W_j denote $W(t_j)$ with $t_j = j\delta t$. Property of Brownian motion tell us that $W_{j+1} = W_j + dW_{j+1}\ for\ j = 0, 1, 2, \cdots, N$, where each dW_j is an independent random variable of the form $\sqrt{\delta t}\ N(0,1)$.
In program 1, we use `R` software to simulate discretized Brownian motion over $[0,1]$ with $N = 500$. Here, the random number generator `rnorm` is used, in fact, `rnorm` produces an independent "pseudorandom" number from the $N(0,1)$ distribution.

Program 1 `# Brownian path simulation`

```
T = 1 # T is the maturity ( time belongs to [0,T])
```