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An approximate method to option pricing in the Heston model

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Abstract
The Heston model is one of the most popular stochastic volatility models for derivatives pricing, and it is a mathematical model describing the evolution of the volatility of an underlying asset. The model proposed by Heston(1993), takes into account non-lognormal distribution of the assets returns, leverage effect and the important mean-reverting property of volatility. In addition, it has a semi-closed form solution for European options. In this paper by means of classical Itô calculus, we decompose option prices as the sum of the classical Black-Scholes formula.This decomposition allows us to develop first- and second-order approximation formulas for option prices and implied volatilities in the Heston volatility framework, as well as to study their accuracy for short maturities. Moreover, we show that the corresponding approximations for the implied volatility are linear(first-order approximation) and quadratic(second-order approximation) in the log stock price.

Keywords and phrases: Stochastic volatility , Heston model, Itô calculus, Black-Scholes formula.

1. Introduction

Stochastic volatility models are a natural extension of the classical BlackScholes model that have been introduced as a way to manage the skew and smiles observed in real market data (see, for example, Hull and White [14], Scott [16], Stein and Stein [15], Ball and Roma [5] and Heston [13]). The study of these models has introduced new important mathematical and practical challenges, in particular related with the option pricing problem and the calibration of the corresponding parameters. In fact, we do not have closed-form option pricing formulas for the majority of the stochastic volatility models, and even in the case when closed-form pricing solutions can be derived (see, for example, Heston [13] or Schbel and Zhu [17]), they do not allow in general for fast calibration of the parameters. A recent trend in the literature has been the development of approximate closedform option pricing formulas. To this end, some authors have presented a perturbation analysis of the corresponding PDE with respect to a specific model parameter, like the volatility (see Hagan et al. [12]), the mean reversion (see Fouque et al. [10] and Fouque et al. [11]) or the