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Numerical Free Vibration Analysis of Higher-Order Shear Deformable Beams

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Abstract

Free vibration analysis of Higher-Order shear deformation beam is studied using a numerical method, namely Differential Transform Method (DTM) which is capable of reducing the size of computational effort and can be applied to various types of differential equations. In this study, the applied DTM is different from those are in the textbook and is just considered as a part of calculation procedure. First, the governing differential equations of beam are derived in a general form considering the shear-free boundary conditions zero shear stress conditions at the top and bottom of a beam. Afterward, using DTM the derived equations governing beams, followed up Higher-Order shear deformation, Timoshenko and Bernoulli-Euler models are transformed to algebraic forms and a set of recurrence formulae is derived. Then, imposing the boundary conditions of the beam at hand, a set of algebraic equations are obtained and expressed in matrix form. Finally, the transverse natural frequencies of the beam are calculated through an iterative procedure. Several numerical examples have been carried out to demonstrate the accuracy and competency of the present method and it is shown that the results obtained by the method are in good agreement with those in the literature. Afterward, the free vibration of beams followed up different models (i.e. Bernoulli-Euler, Timoshenko and different Higher-Order models) are taken into consideration.

Keywords: Classical Beam Theory (CBT); Timoshenko Beam Theory (TBT); Higher-Order Beam Theory (HOBT); Free Vibration; Differential Transform Method (DTM).

INTRODUCTION

Many researchers have used Higher Order Shear Deformable beam model for analysis of beam. First Levinson [1] and Bickford [2] presented a shear deformation theory for rectangular beams called Higher-Order Beam Theory (HOBT). Figure 1(c) schematically shows deformation of a Higher-Order beam. Levinson's third order shear beam theory meets the requirement of the zero shear stress conditions at the top and bottom of a beam. As reported in [3], by using Levinson’s kinematics, Bickford [2] presented a variational consistent sixth-order beam theory and Reddy [4] developed a variational consistent third-order shear plate theory in which the plate kinematics are identical to those of Levinson [1] and then Wang and Wang [5] and Gao and Wang [6] proposed a complicated beam theory that was not able to properly account for the constraints restriction at a clamped end of shear deformable beams .

Differential Transform Method (DTM) is an efficient numerical methods for solution of ordinary and partial differential equations, based on Taylor series expansion of the main variables and coefficients to derive solutions in polynomial form. The concept of one-dimensional DTM was first proposed and applied to solve linear and nonlinear initial value problems in electric circuit analysis by Zho [7].

In this paper, Considering a Higher-Order shear deformation beam, conventional Bernoulli-Euler and Timoshenko beam, one-dimensional Differential Transform Method (DTM) has been employed to derive transverse natural frequencies of the beam in a different way rather than those are in the textbook. Actually, DTM has been considered as a part of calculation procedure. First, the governing equations of a beam are derived in a general form. Then using DTM, the derived equations and boundary conditions of the beam are transformed to a set of algebraic equations and expressed in matrix form. Finally, the unknown transverse natural frequencies of the beam are calculated through an iterative procedure. Several numerical examples have been carried out to prove the competency of the present method and the results have been discussed for the different mentioned models.

Structural Model

Consider a general beam with geometrical and material properties including: the rectangular cross-sectional area A, second moment of inertia I, modulus