Soil Dynamics and Earthquake Engineering 31 (2011) 385–390

Contents lists available at ScienceDirect

Soil Dynamics and Earthquake Engineering

journal homepage: www.elsevier.com/locate/soildyn

# Least square approach to simulate wave propagation in irregular profiles using the indirect boundary element method

A. Rodríguez-Castellanos [a,*], F.J. Sánchez-Sesma [b], C. Ortiz-Alemán [a], M. Orozco-del-Castillo [a]

[a] Instituto Mexicano del Petróleo, Eje Central Lázaro Cárdenas 152, Gustavo A Madero, Mexico DF, Mexico
[b] Instituto de Ingeniería, UNAM, Circuito Escolar S/N, Coyoacán, Mexico DF, Mexico

A R T I C L E I N F O

*Article history:*
Received 26 May 2010
Received in revised form
15 September 2010
Accepted 27 September 2010

A B S T R A C T

In earthquake engineering and seismology it is of interest to know the surface motion at a given site due to the incoming and scattered seismic waves by surface geology. This can be formulated in terms of diffraction of elastic waves and then the indirect boundary element method (IBEM) for dynamic elasticity is used. It is based on the explicit construction of diffracted waves at the boundaries from which they radiate. This provides the analyst with insight on the physics of diffraction. The IBEM has been applied to study the amplification of elastic waves in irregular soil profiles. From the strong or weak satisfaction of boundary conditions and a simple analytical discretization scheme a linear system of equations for the boundary sources is obtained. Here, we explore the use of a weak discretization strategy with more collocation points than force densities. The least squares enforcement of boundary conditions leads to a system with reduced number of unknowns. This approach naturally allows one to use both coarser and finer boundary discretizations for smooth and rapidly varying profiles, respectively. A well studied semicircular canyon under incident P or SV in-plane waves is used to calibrate this method. Several benefits are obtained using mixed meshing that leads to the least squares condensation of the IBEM.



## 1. Introduction

The use of integral equations in engineering and physics can be traced back to the pioneering work of C. Somigliana in 1886. He developed the integral equation that relates displacements to tractions in the boundary of a body using reciprocity [1]. Important developments in this area are the relationships between surface and volume integrals, bear the names of Gauss, Stokes and Green [2]. On the other hand, development of the theory of integral equations is due to Fredholm at the beginning of the 20th century [3].

The most important achievements in the theory of integral equations were not attained until the 1960s when electronic computers became widely available. Significant contributions were made by Kupradze, Mikhlin, Rizzo and Cruse, among many others [4]. A recent overview of boundary integral methods in elastodynamics gives a comprehensive account [5].

In the boundary element method (BEM), the field at any point within a given domain is given in terms of integrals of the field at the boundary of the domain. This means that the totality of the information pertaining to the domain is at the boundary. The BEM is frequently called the boundary integral equations (BIE) method and can be grouped in two families: "direct" that relates the physical variables in a given domain to the values taken by these variables at the boundary and "indirect" that relies on an intermediate unknown, which is usually a distribution of auxiliary sources along the boundary. The BEM and IBEM easily fulfill radiation conditions at infinity and can handle complex boundary geometries. Thus, in most cases they do not require absorbing boundaries.

Early applications of BIE methods in elastodynamics considered the singularities of the integrand kernels, as integrals of the Green functions and its derivatives have to be evaluated at the very location of the sources. The pioneering works by Wong and Jennings [6] and Sills [7] are good examples of these efforts. To avoid singularities, various formulations have been developed [8–18], in which singularities were placed outside the domains of interest. These formulations are early expressions of the method of fundamental solutions (MFS). In the MFS the field is approximated by a linear combination of fundamental solutions expressed in terms of sources located outside the domain of the problem. Its coefficients and the sources' locations are determined by satisfying the boundary conditions in a least squares sense. A comprehensive survey on the MFS and related methods is given by Fairweather and Karageorghis [19]. In strong motion seismology various wave propagation studies for irregular profiles can be regarded as representatives of the MFS as well.

A variational IBEM was presented for dealing with various large problems of site effects [20]. The strategy consisted of constructing a linear representation of the force densities in terms of a complete

* Corresponding author. Tel.: +52 55 91 75 83 25.
*E-mail address:* arcastel@imp.mx (A. Rodríguez-Castellanos).

0267-7261/$ - see front matter © 2010 Elsevier Ltd. All rights reserved.
doi:10.1016/j.soildyn.2010.09.007