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Exact Stiffness and Frequency Relationships for a Doubly Asymmetric Bending-Torsion Thin-Walled Beam

A.Azari Sisi[1], B.Rafezy[2], W.P.Howson[3]
[1]Middle East Technical University, aida.sisi@metu.edu.tr
[2]Sahand University of Technology, rafezyb@sut.ac.ir
[3]Cardiff School of Engineering, howson@cf.ac.uk

ABSTRACT

In this paper the vibration and frequency relationships of the coupled bending-torsional three-dimensional beam with asymmetric cross-section is studied. Firstly, uncoupled bending and torsion vibrations of two-dimensional thin-walled beams are investigated separately and the governing differential equations of motion are solved exactly. This process leads to the bending and torsion dynamic stiffness matrices and the uncoupled natural frequencies are derived. The same procedures are done for the coupled bending-torsion three-dimensional thin-walled beam. Coupled differential equations of motion are developed and solved exactly. Bending-torsion dynamic stiffness matrix is derived as well as the coupled natural frequencies.

It is necessary to solve a transcendental eigenvalue problem to find natural frequencies because the mass of the element is uniformly distributed along the length. This is done using the Wittrick-Williams algorithm.

It is then shown how the coupled natural frequencies are obtained from the corresponding uncoupled values using an exact relationship called relational matrix. This approach presents a simple method for calculation the coupled natural frequencies of the element. Some numerical examples are investigated to verify the theory.

Key Words: Coupled Natural Frequencies, Uncoupled Natural Frequencies, Differential Equations of Motion, Dynamic Stiffness Matrix

1 INTRODUCTION

Coupled bending-torsional vibration of beams has received much attention, typified by [1] , [2] , [3] , [4]. These authors developed the theory using dynamic stiffness method which relates the nodal forces to the corresponding nodal displacements. The coupling between the bending and torsional vibratory modes occures when shear centre and mass centre of the beam cross-section are not coincident. This has been done to the Timoshenko beam members [5], [6]. Existence of axial load has a great influence on the vibration and frequencies of beam elements [7], [8].