Mathematical and Computer Modelling 54 (2011) 1613–1619

Contents lists available at ScienceDirect

Mathematical and Computer Modelling

journal homepage: www.elsevier.com/locate/mcm

# On 4-ordered 3-regular graphs☆

Ming Tsai[a], Tsung-Han Tsai[a], Jimmy J.M. Tan[a], Lih-Hsing Hsu[b,*]

[a] *Department of Computer Science, National Chiao Tung University, Hsinchu, 300, Taiwan ROC*

[b] *Department of Computer Science and Information Engineering, Providence University, Taichung, 43301, Taiwan ROC*

ARTICLE INFO

*Article history:*
Received 8 October 2010
Received in revised form 13 February 2011
Accepted 27 April 2011

*Keywords:*
$k$-ordered
Cubic graphs

ABSTRACT

A graph $G$ is *$k$-ordered* if for any sequence of $k$ distinct vertices $v_1, v_2, \ldots, v_k$ of $G$ there exists a cycle in $G$ containing these $k$ vertices in the specified order. In 1997, Ng and Schultz posed the question of the existence of 4-ordered 3-regular graphs other than the complete graph $K_4$ and the complete bipartite graph $K_{3,3}$. In 2008, Meszaros solved the question by proving that the Petersen graph and the Heawood graph are 4-ordered 3-regular graphs. Moreover, the generalized Honeycomb torus GHT$(3, n, 1)$ is 4-ordered for any even integer $n$ with $n \geq 8$. Up to now, all the known 4-ordered 3-regular graphs are vertex transitive. Among these graphs, there are only two non-bipartite graphs, namely the complete graph $K_4$ and the Petersen graph. In this paper, we prove that there exists a bipartite non-vertex-transitive 4-ordered 3-regular graph of order $n$ for any sufficiently large even integer $n$. Moreover, there exists a non-bipartite non-vertex-transitive 4-ordered 3-regular graph of order $n$ for any sufficiently large even integer $n$.

© 2011 Elsevier Ltd. All rights reserved.

## 1. Introduction

For the graph definitions and notation, we follow the definitions and notation of [1]. Let $G = (V, E)$ be a graph if $V$ is a finite set and $E$ is a subset of $\{(u, v) \mid (u, v)$ is an unordered pair of $V\}$. We say that $V$ is the *vertex set* and $E$ is the *edge set*. Two vertices $u$ and $v$ are *adjacent* if $(u, v) \in E$. A graph is of order $n$ if $|V| = n$. The *degree* of a vertex $u$ in $G$, denoted by $\deg_G(u)$, is the number of vertices adjacent to $u$. A graph $G$ is *$k$-regular* if $\deg_G(x) = k$ for any $x \in V$. A *cubic graph* is a 3-regular graph. A *path* between vertices $v_0$ and $v_k$ is a sequence of vertices represented by $\langle v_0, v_1, \ldots, v_k\rangle$ with no repeated vertex and $(v_i, v_{i+1})$ is an edge of $G$ for every $i$, $0 \leq i \leq k - 1$. We also write the path $\langle v_0, v_1, \ldots, v_k\rangle$ as $\langle v_0, \ldots, v_i, Q, v_j, \ldots, v_k\rangle$ where $Q$ is a path from $v_i$ to $v_j$. A *cycle* is a path with at least three vertices such that the first vertex is the same as the last one.

A graph $G$ is *$k$-ordered* if for any sequence of $k$ distinct vertices $v_1, v_2, \ldots, v_k$ of $G$ there exists a cycle in $G$ containing these $k$ vertices in the specified order. The concept of $k$-ordered graphs was introduced in 1997 by Ng and Schultz [2]. Previous results focus on the conditions for minimum degree and forbidden subgraphs that imply $k$-ordered graphs [3–6]. A comprehensive survey of the results can be found in [6].

In [2], Ng and Schultz posed the question of the existence of 4-ordered 3-regular graphs other than $K_4$ and $K_{3,3}$. In [7], Meszaros solved the question by proving that the Petersen graph and the Heawood graph are 4-ordered 3-regular graphs. Moreover, the generalized Honeycomb torus GHT$(3, n, 1)$ is 4-ordered if $n$ is an even integer with $n \geq 8$. Up to now, all the known 4-ordered 3-regular graphs are vertex transitive. Among these graphs, there are only two non-bipartite graphs,

☆ This work was supported in part by the National Science Council of the Republic of China under Contract 97-2221-E-126-001-MY3.

* Corresponding author.
*E-mail address:* lhhsu@cs.pu.edu.tw (L.-H. Hsu).

0895-7177/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.mcm.2011.04.034